THEORY AND METHODS

Sampling variability of the Kunst-Mackenbach relative index of inequality

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Study objective: To derive methods of calculating confidence limits for the relative index of inequality, defined by Kunst and Mackenbach as a measure of the influence of socioeconomic status on an adverse health index, such as mortality rate. The methods may be used for a health outcome recorded on a continuous scale, as a Poisson count or as a binomial variable.

Results and Conclusion: The confidence limits depend on the sampling variability of both the mean mortality rate and the slope of the regression line of mortality on the socioeconomic status scale variable. The best method for a continuous health outcome is based on Fieller's theorem but a good approximation is obtained by substituting the confidence limits for the slope of the regression line into the formula for the calculation of the index, or by using the variance of the logarithmic transform of the index. The last method is the most appropriate for the construction of significance tests comparing indices. The mortality rates may show statistically significant departure from linearity, while not suggesting that a linear relation is inappropriate, and the main decision is whether to base the confidence limits on the conventional standard error of the slope derived from the regression analysis or whether to use the standard deviation of the estimates of mortality rates.

The relation between health and socioeconomic status is an important topic and a measurement of inequality in health is required. We consider the specific measurement of inequality in mortality, or any other adverse health index, over socioeconomic status, the relative index of inequality (RII), and the construction of confidence limits. Following Pamuk[1] the socioeconomic status categories are ranked on a scale from the lowest to the highest. Each category covers a range on the scale proportional to its population size and is given a value, x, on the scale corresponding to the midpoint of its range. The scale has a range from 0 to 1, so that the lower value of the range of a category is equal to the proportion of the whole population that is of lower socioeconomic status. For each category a measure of mortality y, such as a standardised death rate, is available.

The data are plotted as in figure 1 and a regression line fitted by least squares. Pamuk[1 2] recommended that weighted least squares should be used with the weights proportional to the population size of each category, in order to minimise the effect of deviant rates based on small numbers.

The regression line has the form

$$y = \alpha + \beta x \qquad (1)$$

Pamuk [1 2] defined the relative index of inequality as

$$RII = \frac{-\beta}{\bar{y}} \qquad (2)$$

where $\bar{y}$ is the overall mean death rate. Note that β is negative so that the index is positive.

This index was modified by Kunst and Mackenbach [3 4] as

$$RII_{KM} = \frac{\alpha}{\alpha + \beta} \qquad (3)$$

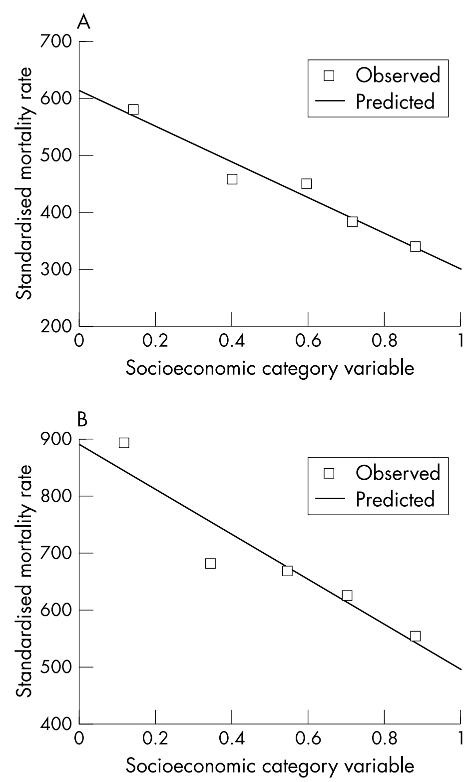

Figure 1 (A) Age standardised mortality rate per 100 000 men in Sydney Statistical Division 1990 to 1994 by quintiles of socioeconomic status. (B) Age standardised mortality rate per 100 000 men in Sydney Statistical Division 1975 to 1979 by quintiles of socioeconomic status.

This index is the ratio of the mortality of the most disadvantaged (x=0) to the most advantaged (x=1). Thus if the index is 1.5 then the mortality rate of the most disadvantaged is 1.5 times as high as that of the most advantaged. Note that the values $x = 0$ and 1 do not correspond to the lowest and highest categories but to the extremes of these categories. They therefore represent extreme, possibly hypothetical, subgroups. In earlier papers Kunst and Mackenbach[5 6] reversed the x scale and defined the index as the proportional increase in mortality of the most disadvantaged relative to the most advantaged; this index is equal to $RII_{KM} - 1$. The RII is sometimes calculated within age groups so inequality is assessed separately for each age group.

Kunst and Mackenbach[3 4] used ordinary least squares, rather than weighted least squares, and we will follow this approach on the grounds that the variability about the regression line may exceed the sampling variability associated with each mortality rate and then the weights give too much weight to the larger categories. It is straightforward to modify the method to use weights if required.

In this paper we consider calculation of the standard error of an estimate of the relative index of inequality. Writing γ for RII_{KM} and noting that $\alpha = \bar{y} - \beta\bar{x}$ then

$$\gamma = \frac{\bar{y} - \beta\bar{x}}{\bar{y} + \beta(1 - \bar{x})} \qquad (4)$$

It is convenient to work with this form as β and $\bar{y}$ are uncorrelated in the least squares estimation. Clearly the variance of γ depends on the variances of β and $\bar{y}$. The variance of γ may be estimated approximately as

$$\mathrm{var}\gamma = \left(\frac{\delta\gamma}{\delta\bar{y}}\right)^2 \mathrm{var}\bar{y} + \left(\frac{\delta\gamma}{\delta\beta}\right)^2 \mathrm{var}\beta$$

This gives

$$\begin{aligned} \mathrm{var}\gamma &= \frac{\beta^2\mathrm{var}\bar{y} + \bar{y}^2\mathrm{var}\beta}{[\bar{y} + \beta(1 - \bar{x})]^4} \\ SE(\gamma) &= \sqrt{\mathrm{var}\gamma} \\ 95\%\ \mathrm{CI} &= \gamma \pm c \times \mathrm{SE}(\gamma) \end{aligned} \qquad (5)$$

where c is the critical 5% value from the appropriate distribution used in the calculation of the variances.

An alternative is to work with the natural logarithm of γ giving

$$\begin{aligned} \mathrm{var}[1\mathrm{n}(\gamma)] &= \frac{\beta^2\mathrm{var}\bar{y} + \bar{y}^2\mathrm{var}\beta}{[\bar{y} - \beta x]^2[\bar{y} + \beta(1 - \bar{x})]^2} \\ \mathrm{SE}[1\mathrm{n}(\gamma)] &= \sqrt{\mathrm{var}[1\mathrm{n}(\gamma)]} \\ 95\%\ \mathrm{CI\ of}\ \gamma &= \exp\{1\mathrm{n}(\gamma) \pm c \times \mathrm{SE}[1\mathrm{n}(\gamma)]\} \end{aligned} \qquad (6)$$

A third method is to use Fieller's theorem[7] to derive confidence limits directly without the intermediate derivation of a standard error. Details are in the appendix.

Because x is in the range 0 to 1 the variance of $\bar{y}$ will be smaller than the variance of β by an order of magnitude. Also β^2 will be smaller than $\bar{y}^2$ provided that γ is less than 3 (this is a very high rate of inequality). Therefore in the expression $\beta^2\mathrm{var}\bar{y} + \bar{y}^2\mathrm{var}\beta$ in equations (5) and (6) the latter term predominates. A fourth method is then to ignore the variability in $\bar{y}$; equation (6) then becomes

$$\mathrm{var}[1\mathrm{n}(\gamma)] = \frac{\bar{y}^2\mathrm{var}\beta}{[\bar{y} - \beta\bar{x}]^2[\bar{y} + \beta(1 - \bar{x})]^2} \qquad (7)$$

Alternatively the limits may be calculated by substituting the limits of β in equation (4); these are the Fieller limits (see equation (11) in appendix).

When it is required to compare two groups an estimate of variance is required and the Fieller limits cannot be used. It is appropriate to work with the logarithmic transform and use equations (6) or (7).

There are two methods of estimating the variances of β and $\bar{y}$. The first is to use the regression output from fitting the regression equation working with the variance based on the standard deviation about the regression line fitted by least squares. This has the disadvantage that there are usually only a few socioeconomic status categories and hence there are few degrees of freedom for the error (in our example, 3 df and $c = t_{3,0.05} = 3.182$) giving an increase in the width of the confidence intervals. Also the death rates that are used within each category have a known accuracy dependent on the sample size from which they have been estimated. This variability contributes to the variability around the regression line but the latter also includes a component because of lack of linearity in the true relation between the death rate and x, or because of a lack of fit of the line for other reasons. The second method is to ignore any lack of linearity or fit and base the variances of β and $\bar{y}$ on the sampling variability of the death rates. Then c is the standardised normal deviate, z=1.96.

An alternative approach is to use Poisson regression. If d is the number of deaths, and the expectation of d, based on the size and age distribution of the group, is m then the Poisson regression is

$$1\mathrm{n}\mu = 1\mathrm{n}m + \alpha + \beta x \qquad (8)$$

where μ is the expectation of d taking account of x. As the death rate is μ/m this model is linear between the logarithm of death rate and x. Then

$$\gamma = \frac{\exp\ \alpha}{\exp(\alpha + \beta)} = \exp(-\beta) \qquad (9)$$

and a confidence interval for γ is obtained directly as exp $[-\beta \pm c \times \mathrm{SE}(\beta)]$.

A second alternative is to use logistic regression, and this is appropriate for data from case-control studies in which the odds ratio (OR) is approximately equal to relative risk. The relative index of inequality, γ, is the odds ratio for x=0 relative to x=1 and equals $\exp(-\beta)$, and a confidence interval is obtained in the same way as for Poisson regression.

It should be noted that when either Poisson or logistic regression is used the RII is obtained directly as the exponential of the slope and the intercept term is not required. For a continuous outcome variable, this is not the case, as the slope is then an absolute measure of difference in health status. Wagstaff[8] refers to this as the slope index of inequality (SII) and notes that it may be interpreted as the absolute effect on health of moving from the lowest socioeconomic group to the highest. A relative measure can only be constructed either by dividing by the overall rate, as in equation (2), or by taking the ratio of the fitted values at x=0 and x=1 to produce the Kunst-Mackenbach measure, equation (3).

EXAMPLE

In a study of inequality in mortality in the Australian state of New South Wales 1975 to 1994, local government areas were

Table 1 Age standardised mortality rate per 100 000 men in the Sydney Statistical Division 1990 to 1994 by quintiles of socioeconomic status

Quintile	x	ASMR (y)	SE (y)
1st (lowest)	0.139	578.5	5.41
2nd	0.401	456.7	4.63
3rd	0.595	449.0	5.90
4th	0.717	381.0	6.31
5th (highest)	0.884	337.4	4.09
Mean	0.547	440.5	5.33*

*Root mean square.

Key points

- The relation between health and socioeconomic status is an important topic.
- The relative index of inequality (RII) is a frequently used summary measure of socioeconomic inequality in health.
- The RII confidence limits depend on two sources of sampling variability, the health outcome measure (such as mortality rate) and the fitted regression line of health outcome on the socioeconomic status scale variable.
- This paper shows that while the best method for deriving confidence limits is based on Fieller's theorem, a very good approximation may be obtained using a simpler method.

sorted into quintiles based on a measure of social disadvantage. For each five year period by gender the relative index of inequality was used to summarise the relation between the age standardised mortality rate (ASMR, y) and the quintile cumulative population proportions (x). The data in table 1 and figure 1(A) are for men in 1990–1994. The standard errors shown for each ASMR were calculated assuming a Poisson distribution of the number of deaths in each age group. β is estimated as −310.1 and γ as 2.033. The 95% confidence limits of γ are given in table 2.

Within the columns of table 2 the limits are similar for all the methods except for those calculated using SE(γ). The limits calculated using the limits of β are only slightly narrower than those based on Fieller's method as may be predicted as $\bar{y}^2\text{var}\beta$ is 30 times $\beta^2\text{var}\bar{y}$ so that ignoring the latter has little effect. The limits using the standard deviation from estimation of the age standardised mortality rates are much narrower than those using the standard deviation about the regression line. This is mainly because the latter standard deviation is 4.3 times as great as the former and also because of the effect of using a t value on 3 df of 3.18 instead of a z value of 1.96. A formal test of linearity is the ratio of the variances ($22.9^2/5.33^2 = 18.5$ as a F with 3 and ∞ df, $p < 0.001$).

Figure 1(B) shows similar data for 1975–1979. The index is estimated as 1.81 with 95% confidence interval using equation (7) of 1.18 to 2.79 including non-linearity, or 1.74 to 1.88 ignoring non-linearity. The former are very wide as a consequence of the wide variability about the best fitting line of the two lowest points on the socioeconomic status scale. Suppose it is required to test whether there has been a change in the index between 1975–1979 and 1990–1994. Then taking account of non-linearity var(lnγ) may be recalculated for each period using the combined estimate of the variance about the fitted lines that is 1604 with 6 df. It should be noted that although the separate estimates (2684 for 1975–79 and 524 for 1990–1994) differ by a factor of 5 this is well within the bounds of chance when each is estimated on only 3 df. Then the test statistic is calculated as the difference between the two values of lnγ divided by the square root of the sum of their variances, which gives 0.59 as a t statistic with 6 df. Clearly the result is not significant as is clear from the wide confidence intervals. If the non-linearity is ignored then using the variances calculated from equation (7) for each period the test statistic is 3.68 as a z statistic giving a highly significant effect as is again clear from the confidence intervals.

DISCUSSION

The measurement of inequalities in health is an important topic and the relative index of inequality is a frequently used measure.[8–17] Poisson regression[9–12] or logistic regression[12–17] are often used, while analysis of a continuous health outcome seems to be less common. Hence it is necessary to be able to estimate the sampling variability of an estimate of this measure in order to produce confidence intervals and to compare two measures.

The Fieller method of calculating the confidence limits may be expected to be superior as it takes account of the variability in both β and $\bar{y}$, and does not assume that either γ or its logarithm are normally distributed. However, in practice the limits obtained by substituting the limits of the regression coefficient β in the calculation of γ and ignoring the variability in the estimation of the mean mortality rate are a very good approximation. Another reason for basing the limits only on varβ is so that the 95% confidence limits and the 5% significance test will be consistent, as a significance test of the index is obtained by testing β. An alternative to Fieller's theorem is to calculate limits using SE[ln(γ)] and this is the best method for estimating the sampling variability for the calculation of a test statistic to compare two groups. Kunst and Mackenbach[5 6] gave confidence intervals of the index which were based on the sampling variability of β. As they used Poisson regression methods exp(−β) gave the index and $\bar{y}$ was uninvolved.

The main decision is which standard deviation to use in the calculation of the limits. The conventional approach is to use the standard deviation about the regression line. This would certainly be the advised course if an assumption of a real linear relation between the mortality rate and the measure of socioeconomic status was considered essential. However, there is no a priori reason to suppose that the relation is necessarily linear and the fitting of a straight line may then be regarded as a convenient approximation to the general trend. It is then more appropriate to ignore the non-linearity and use the standard deviation calculated from the original estimation of

Table 2 95% confidence limits calculated by different methods

Method of deriving limits	SD (22.9) from regression fit and c=3.182 (t on 3 df) (var$\bar{y}$=104.8, varβ=1576)	SD (5.33) from estimation of ASMRs and c=z =1.96 (var$\bar{y}$=5.68, varβ=85.4)
Fieller's method (eqn 10)	1.512 to 2.828	1.946 to 2.126
Using limits of β (eqn 11)	1.514 to 2.796	1.947 to 2.124
Using SE(γ) (eqn 5)	1.405 to 2.661	1.943 to 2.123
Using SE[ln(γ)] (eqn 6)	1.493 to 2.769	1.945 to 2.125
Ignoring var($\bar{y}$) (eqn 7)	1.501 to 2.755	1.947 to 2.124

the mortality rates. When fitting a Poisson regression the choice is between estimating the variance of β under the Poisson distribution (ignoring non-linearity if present) or inflating the variance to take account of extra Poisson dispersion due to non-linearity. The latter situation is termed "over-dispersion" (see McCullagh and Nelder[18]).

In the example there was clear evidence of excess variability about the fitted line and yet figure 1 suggests that linear relations are the most appropriate relations that could be used. The lack of fit may be attributable to non-linearity in the true relation or to imprecision in defining the socioeconomic status variable. In the former case it seems appropriate to ignore the non-linearity, as it is a systematic rather than a random effect, whereas in the latter the lack of fit indicates random variability that should be taken into account. It is conservative to base the limits on the standard deviation about the regression when assessing whether changes in inequality have occurred over time.

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APPENDIX
Fieller's method

$$\gamma = \frac{\bar{y} - \beta\bar{x}}{\bar{y} + \beta(1-\bar{x})}$$

Define

$$\begin{aligned} Z &= \bar{y} - \beta\bar{x} - \gamma[\bar{y} + \beta(1-\bar{x})] \\ &= \bar{y}(1-\gamma) - \beta(\bar{x} + \gamma - \gamma\bar{x}) \end{aligned}$$

Then

$$\text{var}Z = (1-\gamma)^2\text{var}\bar{y} + (\bar{x} + \gamma - \gamma\bar{x})^2\text{var}\beta$$

The best estimate of γ is when z=0 and by Fieller's theorem the confidence limits for γ are the solutions of the equation

$$\frac{Z}{\text{SE}(Z)} = \pm c$$

where c is the critical value of the appropriate distribution. Therefore the confidence limits for γ are the solutions of

$$\frac{\{\bar{y} - \beta\bar{x} - \gamma[\bar{y} + \beta(1-\bar{x})]\}^2}{(1-\gamma)^2\text{var}\bar{y} + (\bar{x} + \gamma - \gamma\bar{x})^2\text{var}\beta} = c^2$$

This is a quadratic equation in γ and the two roots are the lower and upper confidence limits.

After some algebraic manipulation the limits are found to be

$$\gamma_{\text{CL}} = \frac{B \pm \sqrt{D}}{A} \qquad (10)$$

where

$$A = [\bar{y} + \beta(1-\bar{x})]^2 - c^2[\text{var}\bar{y} + (1-\bar{x})^2\text{var}\beta]$$
$$B = (\bar{y} - \beta\bar{x})[\bar{y} + \beta(1-\bar{x})] - c^2[\text{var}\bar{y} - \bar{x}(1-\bar{x})\text{var}\beta]$$

and

$$D = c^2[\beta^2\text{var}\,\bar{y} + \bar{y}^2\text{var}\,\beta - c^2\text{var}\,\bar{y}\,\text{var}\beta]$$

If the variance of $\bar{y}$ is small relative to $\bar{y}^2\text{var}\beta/\beta^2$ then the limits may be obtained by putting var $\bar{y}$ equal to zero in the above to give as limits

$$\gamma_{\text{CL}} = \frac{\bar{y} - [\beta \pm c\text{SE}(\beta)]\bar{x}}{\bar{y} + [\beta \pm c\text{SE}(\beta)](1-\bar{x})} \qquad (11)$$

That is, the limits for γ are obtained by substituting the limits of β in the calculation of γ (equation 4).

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REFERENCES
1 **Pamuk ER**. Social class inequality in mortality from 1921 to 1972 in England and Wales. *Population Studies* 1985;**39**:17–31.
2 **Pamuk ER**. Social-class inequality in infant mortality in England and Wales from 1921 to 1980. *European Journal of Population* 1988;**4**:1–21.
3 **Kunst AE**, Mackenbach JP. *Measuring socio-economic inequalities in health*. Copenhagen: World Health Organisation, 1995.
4 **Mackenbach JP**, Kunst AE. Measuring the magnitude of socio-economic inequalities in health: an overview of available measures illustrated with two examples from Europe. *Soc Sci Med* 1997;**44**:757–71.
5 **Kunst AE**, Mackenbach JP. International variation in the size of mortality differences associated with occupational status. *Int J Epidemiol* 1994;**23**:742–50.
6 **Kunst AE**, Mackenbach JP. The size of mortality differences associated with educational level in nine industrialized countries. *Am J Public Health* 1994;**84**:932–7.
7 **Fieller EC**. A fundamental formula in the statistics of biological assay, and some applications. *Q J Pharmacol* 1944;**17**:117–23.
8 **Wagstaff A**, Paci P, van Doorslaer E. On the measurement of inequalities in health. *Soc Sci Med* 1991;**33**:545–57.
9 **Marmot MG**, Shipley MJ. Do socioeconomic differences in mortality persist after retirement? 25 year follow up of civil servants from the first Whitehall study. *BMJ* 1996; **313**:1177–80.
10 **Davey Smith G**, Hart C, Hole D, *et al*. Education and occupational social class: which is the more important indicator of mortality risk? *J Epidemiol Community Health* 1998; **52**:153–60.
11 **Marang-van de Mheen PJ**, Davey Smith G, *et al*. Socioeconomic differentials in mortality among men within Great Britain: time trends and contributory causes. *J Epidemiol Community Health* 1998;**52**:214–18.
12 **Mackenbach JP**, Kunst AE, Cavelaars AE, *et al*. Socioeconomic inequalities in morbidity and mortality in western Europe.*Lancet* 1997;**349**:1655–9.
13 **Cavelaars AE**, Kunst A, Geurts JJ, *et al*. Differences in self reported morbidity by educational level: a comparison of 11 western European countries. *J Epidemiol Community Health* 1998; **52**:219–27.
14 **Regidor E**, Dominguez V, Navarro P, *et al*. The magnitude of differences in perceived general health associated with educational level in the regions of Spain. *J Epidemiol Community Health* 1999;**53**:288–93.
15 **Eachus J**, Chan P, Pearson N, *et al*. An additional dimension to health inequalities: disease severity and socioeconomic position. *J Epidemiol Community Health* 1999;**53**:603–11.
16 **Drummer TJB**, Dickinson HO, Pearce MS, *et al*. Stillbirth risk with social class deprivation: no evidence of increasing inequality. *J Clin Epidemiol* 2000;**53**:147–55.
17 **Anitua C**, Esnaola S. Changes in social inequalities in health in the Basque Country. *J Epidemiol Community Health* 2000;**54**:437–43.
18 **McCullagh P**, Nelder JA. Generalized linear models. 2nd edn. London: Chapman and Hall, 1989:124–8.