of two Poisson variables had previously been shown by Kahn, Brownlee, and Armitage.

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References


Professor Liddell replies:

That I did not discuss the comparison of two SMRs was not because I considered the matter unimportant. On the contrary, it merits much more than the four journal pages I needed to explain fully the exact analysis of a single SMR (≡D/E) on just one pair of assumptions: that D was generated by a Poisson process; and that E was without sampling error.

If \( N_i = \alpha_i \cdot N \) and \( n_i = \beta_i \cdot n \) (where \( \sum \alpha_i = \sum \beta_i = 1 \)) are the numbers of subject-years (or of persons) in the i-th age group of the reference and study populations, and if \( x_i \) is the number of deaths in the i-th age group of the reference population, then \( E \) is the total over age groups of the terms \( n_i (x_i / N_i) \). Thus \( E = (n/N) \cdot \sum (\beta_i / \alpha_i) x_i \), and \( \text{var}(E) = (n/N)^2 \cdot \sum (\beta_i / \alpha_i)^2 x_i \) which (for \( N > n \)) is smaller than \( E \) by a factor of roughly \( (n/N) \), and becomes negligible for large \( N \). With national or state or provincial populations frequently used for reference, \( E \) can indeed be taken as without sampling error.

In this context, the statement of Frentzel-Beyme is irrelevant; my use of the word “misleading” was unfortunate, and I apologise to all concerned.

Nevertheless, precisely how to set limits on a single SMR “yet taking both numerator and denominator variation into account in circumstances where \( E \) cannot be considered as fixed" is more complex than can be dealt with in a brief reply. Suffice it to say here that the methods of Ederer and Mantel would give exact limits only if \( \beta_i = \alpha_i \) for all i. This would not be true, if only because of ageing, even in comparing the same population in successive years.

As to my own recommendations, they are to use the exact method in every situation where the two assumptions are valid. For \( D < 15 \), standard tables of the \( \chi^2 \) distribution cover all usual requirements. For \( 5 < D < 15 \) where required values of \( \chi^2 \) are not tabulated, and for \( D > 14 \), my equations (1a), (2a), (3a), and (4a) based on the Wilson/Hilferty transformation, are highly reliable.

Finally, may I take this opportunity of pointing out a typographical error in my article? On page 87, the expression “bd=1” in line 26 of the right-hand column should read “D=1”.

Letter