STUDIES ON PUBERTY

PART II. THE PATTERN OF DIFFERENTIAL GROWTH*

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1. INTRODUCTION

Previous investigations concerning growth in the human subject are mainly restricted in scope to consideration of increments in height, in weight, and less often in more specific measurements, related to a chronological time-scale. There has been little attempt to interpret such anthropometric material against the background of accredited principles which have been established for other species by the method of experimental embryology.

The importance of regional differential growth as the essential feature of the gross morphogenetic, as opposed to histogenetic, aspect of development has been variously emphasized, notably by Child (1915). The study of continuous changes in body proportions from foetus to adult was also brought into focus by D'Arcy Thompson† (1917), who pointed out that the growth intensity of the body conforms with a more or less regular pattern, the relative growth-ratios of parts of the body being defined by an orderly system of growth gradients. Anthropometric investigation need not, therefore, be fruitless if directed to evaluating how relative growth of different dimensions during particular phases of development contributes to changes of bodily shape characteristic of the sexes. In this way, we may bring into focus problems which await interpretation in terms of our advancing knowledge of cell metabolism.

Part I of the present series (Hogben, Waterhouse, and Hogben, 1948) explored certain features of differential growth in relation to qualitative changes taking place at puberty, indicating specific hormonal influences on three sexually-differential indices of body build, viz. bispinous width of pelvis, bisacromial width of shoulders, and neck girth. It is now

our purpose to analyse in greater detail, and from a new viewpoint, such measurable trends of differential growth during the human sexual metamorphosis.

Several writers, notably J. S. Huxley (1924; 1932; 1945), have studied allometric growth (i.e. disproportionate growth of parts of the body, resulting in change of body-shape with age) more especially with respect to the development of Crustacea and holometabolous insects. Huxley's approach to the concept of relative growth envisages the development of the organism as a succession of discrete phases during which growth along one dimension relative to another proceeds with comparative uniformity in accordance with an exponential law, such phases being commonly few in number and long in duration. If we consider two such anthropometric measurements as arm length and standing height, the empirical postulate of the allometric attack on morphogenesis is that for any particular period of development arm length (x_a) increases in relation to standing height (x_h) in accordance with an equation of the form

$$x_a = k(x_h)^b \quad (i)$$

$$\log x_a = b \log x_h + \log k \quad (ii)$$

By differentiating (ii)

$$\frac{dx_a}{x_a} = b \frac{dx_h}{x_h}$$

$$\text{and} \quad \frac{dx_a}{dx_h} \cdot \frac{x_h}{x_a} = b \quad (iii)$$

Now a proportionate *specific growth rate*, for any site or organ, may be defined as the increment in size of that organ, in unit time, per unit size. If A represents the size of an organ at one instant, and $A + \delta A$ its size after an interval of time δt , the specific growth ratio (g) would be:

$$g = \frac{\delta A}{A} \cdot \frac{1}{\delta t} = \frac{1}{A} \cdot \frac{\delta A}{\delta t};$$

And if $a = \log A$,

$$g = \frac{\delta a}{\delta t} \approx \frac{da}{dt}$$

* Part I appeared in this Journal in January, 1948.

† This author drew attention to the possibility of studying growth in terms of co-ordinate transformations, a suggestion followed up in Medawar's contribution (1944) on measurements of the human foetus. From a study of amphibian metamorphosis, one may well suspect that the orderly pattern disclosed by so admittedly attractive a schematization deflects atention from specifically localized foci of differential growth, severally inviting the evaluation of a specific responsible agency; in short, the very simplicity of the relations exhibited by this method may conceal the complexity of the machinery we are investigating.

If the relationship between two such logarithmic growth rates is approximately linear, so that

$$a_1 = ba_2 + c;$$

then $\log A_1 = b \log A_2 + c$,

and $A_1 = CA_2^b$ where $\log C = c$,

and the slope is $b = \frac{da_1}{da_2} = \frac{da_1}{dt} \div \frac{da_2}{dt} = \frac{g_1}{g_2}$.

A phase of relative growth is thus definable by the numerical value of the constant b , denoted as the *differential growth coefficient* (or *ratio*) for the two dimensions. An important implication of (ii) is that we have eliminated time as a co-ordinate for observation and are subsequently concerned only with *relative growth* of different dimensions. The justification for this approach is, of course, entirely empirical, and, as such, calls for validation in any particular investigation such as the present. Our reasons for thus experimentally adopting the method are: (a) that it is reasonable to suppose that growth follows an exponential law, which is best illustrated by using logarithms of measurements; (b) that with such use of logarithms the allometric equation reduces to that of a straight line, a form which, if permissible, facilitates interpretation of results. This approach appears to be a good empirical approximation herein, as may be judged from the graphs included below.

In examining the relevance of the method to a study of human sex differentiation, we require a more exacting analytical technique. Huxley's development of the theme, competent within the domain of invertebrate growth and form as first used, involves a legitimate crudity in that it relies on intuition to arbitrate on the choice of a standard axis of reference, i.e. the abscissa, defined by Huxley merely as a measure of growth of the body in general as opposed to growth of a specific part, represented by the ordinate. There is no *a priori* reason why the growth rates in the two sexes of height, weight, or any other particular measurement should tally, so we must first make a direct investigation of sex differentials at different sites in order to establish which, if any, exhibit *sexually invariant* growth rates during puberty.

The present investigation was conducted on records of a single examination of each individual in a sample comprising 642 boys and 662 girls drawn from a sequence of half-yearly age groups from 8½ to 17 years. This plan allows direct observation and yields age-means of metric indices as well as the time of first appearance and duration of development of non-metric features of sexual maturation. The sample was taken from primary and secondary

schools in the borough of Tottenham. Data were recorded during routine school medical examinations and comprised, in addition to the qualitative attributes of sexual development recorded and discussed in the preceding publication, the following dimensions:

1. Weight
2. Height
3. Sitting height
4. Trunk length (suprasternal notch to pubic symphysis)
5. Trunk depth (nipple level)
6. Trunk width (nipple line)
7. Thorax circumference: (a) at nipple, (b) at xiphoid process
8. Abdominal circumference (umbilicus)
9. Bisacromial width of shoulders
10. Bispinous width of pelvis* (posterior superior iliac spines)
11. Girth of neck (level of thyroid cartilage)
12. Arm length (acromion to 1st carpal crease)
13. Hand length (1st carpal crease to tip of 3rd finger)
14. Leg length (anterior superior iliac spine to external malleolus)
15. Cephalic length
16. Cephalic breadth
17. Bizygomatic width

For our present purpose we are concerned only with average measurements representative of selected somatic features as above, for each age group. The range of variability to which the individual is subjected is a separate problem, reserved for further communication. Here we restrict ourselves to an overall picture of a *composite* individual, himself or herself a fiction, legitimate as a summary of a more or less homogeneous sample.

2. SEXUAL HOMOPLASIA AND HETEROPLASIA

In searching for a sexually invariant or *homoplastic* dimension (i.e. one characterized by the same proportionate growth rate for both sexes throughout the age-range studied) in contradistinction to the state of *heteroplasia*, our procedure has been to plot on a logarithmic grid successive mean values of particular dimension A for girls as ordinates against the corresponding mean values for boys of the same half-year group. We have then found it possible to give close representation of the distribution by fitting one or more straight lines, the parameters of which define the trend of relative growth during the relevant phase of development. If A_f and A_m

* We take this opportunity of drawing attention to an ambiguity in the title of two text figures of Part I (January, 1948), in which the expression *pelvic* width might suggest bicristal as opposed to bispinous width.

respectively specify a measurement for girls and boys of the same age group,

$$\text{then} \quad \log A_f = l \log A_m + c, \quad (i)$$

$$\text{i.e.} \quad A_f = C(A_m)^l$$

$$\text{and} \quad \frac{dA_f}{A_f} = l \cdot \frac{dA_m}{A_m} \quad (ii)$$

l and C respectively denote the *sexually differential growth coefficient* and the *sexually differential growth constant*.

Typically, the graphs were sigmoidal, showing a three-phase inter-relationship, but occasionally there were two linear trends or only one. The usual method of curve-fitting for a series of points having a linear trend is by the determination of one of two regression coefficients by the method of least squares. This method assumes dependence of one variate upon the other by minimizing either the abscissal or the ordinate differences, and is therefore unsuitable in situations such as the present in which we are seeking the relationship between two variates which are logically independent, although concurrent. We have therefore used for curve-fitting throughout this study a technique of *least square-perpendiculars*, giving the line of closest fit to all the points. A detailed account of the rationale of this method and of appropriate measures of goodness-of-fit will be the subject of a subsequent communication.

Fig. 1 (overleaf) shows the sigmoidal three-phase growth relationship typical of most dimensions, and the deviations from this plan characteristic of atypical dimensions. Table I (overleaf) demonstrates the relevant parameters for each dimension for each discrete phase of growth, and for the whole eight years, together with the goodness-of-fit criteria. The ratio of the residual variance of the points about the fitted line to their variance about a line at right angles may be expressed as $(1 - R^2)$ where R is an analogue of the product-moment correlation coefficient, $R^2 = 1$ signifying a perfect fit. A second criterion of reliability shows the justification for fitting two or three separate lines to the distribution: Σp^2 , the residual variance of the points representative of a particular growth-epoch about the line which exclusively summarizes that epoch, is expressed as a ratio to the residual variance, ΣP^2 , of the same points about a line specifying growth over the whole period. In our use of these two ratios as goodness-of-fit criteria, they are best expressed as percentages,

$$\text{i.e. } 100(1 - R^2) \text{ and } 100 \frac{\Sigma p^2}{\Sigma P^2}.$$

The product-moment correlation coefficient, also tabulated, is a general summarizing index of concurrence between the two sexes, but has no exact significance here.

By recourse to this procedure, we can define the typical pattern of growth in females relative to males in chronological terms, since each point on the charts refers to a particular age. At an age varying from 10.75 to 11.75, the female-male growth ratio in most dimensions undergoes a striking change in favour of the female, and 2 to 2½ years later there is a second equally abrupt change in the opposite direction. From these observations *per se*, it would be illegitimate to draw conclusions respecting the sex-specificity of either change. Comparison with our previously published observations on parallel changes taking place in the same sample, however, brings to light a striking analogy. At 11.25 years half the girls in the sample are exhibiting the first qualitative signalization of puberty, the development of the mammary glands. It is not until two years later, at 13.25, that half the boys in the sample show the first signs of the earliest male characteristic, pubic hair. In view of the closeness of the distribution to symmetry, there will be little difference between the median age and the mean age of appearance of the sexual metamorphosis. We have in fact strong grounds for associating the inception of visible qualitative manifestations of a new phase of endocrine activity in girls at 11.25 years and in boys at 13.25 years, with the discontinuities in the relative growth rates of boys and girls at most sites at an age from 10.75 to 11.75 and again between 13.25 and 14.25 years, confirming the suspected existence of a growth spurt at the beginning of puberty.

Such a pattern of growth, characteristic of females relative to males, holds good for weight, height, sitting height, trunk length, thorax circumference (at nipple and at xiphoid process), abdominal circumference, arm length (with and without hand), leg length, trunk depth, and neck girth. Certain other dimensions do not conform to this plan. Trunk width (Fig. 1d) and bisacromial width (Fig. 1f) can be resolved into two lines only; stimulation of growth rate appears to be confined to the male. Bispinous width discloses an arresting picture (Fig. 1g) indicative of striking sexual heteroplasia: the female pelvis evidently grows more rapidly after the onset of puberty and the male pelvis more slowly, since the slope increases at the time of onset of puberty in each sex. A difference not emphasized in Part I of this series is that cephalic length (Fig. 1h) stops increasing in the pubescent female, whilst cephalic breadth (Fig. 1i) exhibits no stimulation at puberty in either sex. A third cephalic measurement taken, bizygomatic width, apparently undergoes more random variation than any other feature studied.

FIG. 1.—Growth of girls relative to boys illustrated by logarithmic plotting of corresponding age-means.

TABLE I
PHASES OF SEXUALLY DIFFERENTIAL GROWTH

Dimension	Age (years)	r	l	C	$100(1-R_l^2)$	$100(\Sigma P^2 \div \Sigma P^2)_l$
Weight	8.75-16.75	0.964	0.9149	1.489	1.93	
	8.75-10.75	0.864	0.4499	9.743	4.37	17.09
	10.75-13.25	0.972	2.2624	0.0041	2.65	4.48
	13.25-16.75	0.927	0.4801	11.41	2.39	22.45
Height	8.75-16.75	0.980	0.7418	3.615	0.87	
	8.75-11.25	0.950	0.6707	5.069	2.09	41.28
	11.25-13.25	0.958	1.5123	0.0794	2.28	8.34
	13.25-16.75	0.964	0.4340	17.38	1.08	15.46
Sitting height	8.75-16.75	0.950	0.8656	1.791	2.50	
	8.75-11.25	0.889	0.8006	3.070	2.45	45.95
	11.25-13.25	0.927	2.0147	0.0026	3.02	8.11
	13.25-16.75	0.833	0.4424	11.70	5.16	39.51
Trunk length	8.75-16.75	0.966	0.9817	1.074	1.71	
	8.75-11.25	0.801	0.6837	3.262	9.85	6.15
	11.25-13.25	0.925	2.1799	0.0107	2.33	14.99
	13.25-16.75	0.941	0.5108	6.834	2.18	19.00
Thorax circumference (nipple)	8.75-16.75	0.963	1.1623	0.4921	1.75	
	8.75-11.75	0.954	0.9774	1.059	2.44	39.11
	11.75-13.75	0.948	2.4063	0.0025	1.22	6.01
	13.75-16.75	0.817	0.5242	8.130	6.71	36.40
Thorax circumference (xiphoid)	8.75-16.75	0.938	0.8720	1.651	3.23	
	8.75-11.75	0.883	1.1753	0.4641	6.24	63.53
	11.75-13.75	0.814	2.1647	0.0072	6.60	20.76
	13.75-16.75	0.706	0.2678	22.57	5.95	23.20
Abdominal circumference	8.75-16.75	0.943	0.8975	1.496	3.16	
	8.75-10.75	0.865	0.2846	17.06	0.96	8.87
	10.75-13.25	0.920	2.3562	0.0041	2.47	10.44
	13.25-16.75	0.822	0.3594	14.08	4.74	26.18
Total arm length	8.75-16.75	0.970	0.7593	2.589	1.88	
	8.75-10.75	0.935	0.6254	4.439	2.69	56.75
	10.75-13.25	0.863	1.4100	0.0172	8.58	38.81
	13.25-16.75	0.891	0.3866	12.94	3.03	27.55
Arm length	8.75-16.75	0.960	0.7771	2.187	1.96	
	8.75-10.75	0.950	0.5270	5.577	1.51	74.00
	10.75-13.25	0.772	1.7398	0.0525	10.21	39.42
	13.25-16.75	0.921	0.3294	13.25	1.82	9.79
Hand length	8.75-16.75	0.945	0.7195	2.240	2.35	
	8.75-10.75	0.804	0.9125	1.297	10.78	37.82
	10.75-13.25	0.979	1.3711	0.3642	1.10	3.71
	13.25-16.75	0.933	0.2891	7.949	0.41	7.94
Leg length	8.75-16.75	0.959	0.7030	3.667	1.72	
	8.75-10.75	0.939	0.7782	2.625	2.71	34.77
	10.75-13.25	0.906	1.2346	0.3712	4.72	27.89
	13.25-16.75	0.911	0.2674	25.84	2.81	9.91
Trunk depth	8.75-16.75	0.934	0.8716	1.348	3.39	
	8.75-11.25	0.853	0.4523	4.137	4.85	19.43
	11.25-13.75	0.896	2.5255	0.0140	2.57	8.10
	13.75-16.75	0.646	0.3357	6.401	10.21	43.76
Neck girth	8.75-16.75	0.944	0.5913	3.735	2.36	
	8.75-11.75	0.720	0.3541	8.110	7.57	37.52
	11.75-13.75	0.880	1.5747	0.1390	5.36	13.08
	13.75-16.75	0.842	0.2671	11.54	3.06	24.88
Bispinous width	8.75-16.75	0.911	1.6558	0.2587	3.80	
	8.75-11.75	0.730	0.5368	2.548	6.12	34.11
	11.75-14.25	0.799	1.9569	0.1338	0.38	46.58
	14.25-16.75	0.201	6.7384	4.6(10 ⁻⁶)	26.94	70.73
Trunk width	8.75-16.75	0.931	0.3997	5.303	1.49	
	8.75-13.25	0.823	0.5101	3.824	6.68	89.63
	13.25-16.75	0.750	0.2716	7.852	5.02	78.69
Bisacromial width	8.75-16.75	0.923	1.0660	0.8960	4.08	
	8.75-13.25	0.939	2.0857	0.0321	1.73	18.03
	13.25-16.75	0.875	0.4831	6.510	4.05	25.75
Cephalic breadth	8.75-16.75	0.789	0.4896	4.407	8.19	
	8.75-13.25	0.844	1.3962	0.3095	7.71	10.91
	13.25-16.75	-0.0576	-0.0080	17.71	10.95	0.89
Bizygomatic width	8.75-16.75	0.865	0.9260	1.174	7.26	—

 r =product-moment correlation coefficient. $(1-R_l^2)$ and $(\Sigma P^2 \div \Sigma P^2)_l$ =goodness-of-fit criteria. l =sexually differential growth coefficient. C =sexually differential growth constant.

We must now compare directly the pre- and post-pubescent growth patterns in the two sexes. By what is mathematically equivalent to a change of origin, we can make comparisons between the dimensions of boys and girls at corresponding developmental or *pubertal* ages, the latter being here defined as *age with respect to the onset of puberty* (Table II). In accordance with conclusions set forth in Part I of this series, and confirmed by the present method, our average estimates for qualitative and quantitative signalization of puberty—our so-called *critical ages*—are 11.25 and 13.25 years for girls and boys respectively. We therefore plot for each dimension the mean for girls aged x years as ordinates against the corresponding mean for boys aged $(x+2)$ years, and obtain a result comparatively undistorted by differential age at onset of pubertal acceleration.

TABLE II
CHRONOLOGICAL RELATIONSHIPS OF PUBERTAL AGE

Group No.	Pubertal Age (years)	Chronological Age (years)	
		Boys	Girls
1	-2.5	10.75	8.75
2	-2.0	11.25	9.25
3	-1.5	11.75	9.75
4	-1.0	12.25	10.25
5	-0.5	12.75	10.75
6	0	13.25	11.25
7	+0.5	13.75	11.75
8	+1.0	14.25	12.25
9	+1.5	14.75	12.75
10	+2.0	15.25	13.25
11	+2.5	15.75	13.75
12	+3.0	16.25	14.25
13	+3.5	16.75	14.75

Fig. 2 (opposite) shows the results of the application of this correction. For we can write:

$$\log A_f = v \log A_{m+2} + k$$

$$A_f = K(A_{m+2})^v$$

and

$$\frac{dA_f}{A_f} = v \cdot \frac{dA_{m+2}}{A_{m+2}} \quad (\text{iii})$$

v and K respectively denote the *standardized sexually differential growth coefficient* and the *standardized sexually differential growth constant*, and are exhibited for each phase of relative growth in Table III (overleaf). In this series there is no discernible discontinuity in the trend of the growth rate of girls relative to boys with respect to weight, height, sitting height, trunk length, trunk width, thorax circumference (at nipple and at xiphoid process), abdominal circumference, arm length, and

leg length. Trunk depth and neck girth display a relative increase of female growth, at approximately eighteen months and six months respectively after the estimated age of inception of puberty. Bisacromial width and cephalic length and breadth (Figs. 1f, h, and i) undergo a relative spurt in boys, and bispinous width (Fig. 1g) exhibits a striking change in the opposite direction at the same time. The scatter for each phase of growth is shown as before both graphically and by the goodness-of-fit criteria.

We now have two estimates of the course of relative growth in males and females based respectively on chronological and pubertal age. Our intention is not to impute conformity of growth with a particular mathematical formulation, but to group and summarize the data conveniently. Accordingly, our necessary conditions for sexual homoplasia may be defined as:

- (1) (a) Exhibition of the characteristic sigmoidal or triphasic pattern of relative growth when mean observations paired on a chronological basis are plotted on a logarithmic scale.
- (b) Equivalence of the net growth rate in the two sexes with reference to the whole age-range.

$$\text{i.e. } \frac{dA_f}{A_f} = \frac{dA_m}{A_m}, \text{ and } l=1 \quad (\text{cf. equation (ii)})$$

- (2) (a) Exhibition of a continuous rectilinear relationship when boy-girl means are paired on a development age scale centred at the start of puberty in each sex.

- (b) For the whole range of this relationship,

$$\frac{dA_f}{A_f} = \frac{dA_{m+2}}{A_{m+2}}, \text{ i.e. } v=1 \quad (\text{cf. equation (iii)}).$$

- (3) Satisfaction of the goodness-of-fit criteria,

$$R^2 \text{ and } \frac{\sum p^2}{\sum P^2}$$

Since conditions (1) *b* and (2) *b* are the salient numerical desiderata of conformity between male and female growth-patterns, it is desirable to rank all dimensions satisfying (1) *a* and (2) *a* with respect to each of these. For arranging dimensions in order of equivalence of male and female growth rates, computation of $(l-1)$ and $(v-1)$ immediately suggests itself. We can therefore write:

$$l-1 = \left(\frac{dA_f}{A_f} \div \frac{dA_m}{A_m} \right) - 1$$

$$= \frac{\frac{dA_f}{A_f} - \frac{dA_m}{A_m}}{\frac{dA_m}{A_m}} \quad (\text{iv})$$

$$= \frac{\text{Excess growth rate of females over males}}{\text{Male growth rate}}$$

STUDIES ON PUBERTY PART II

FIG. 2.—Growth of girls relative to boys of the same pubertal age illustrated by logarithmic plotting of age-means for corresponding pubertal ages.

TABLE III
PHASES OF SEXUALLY DIFFERENTIAL GROWTH
(DEVELOPMENTAL BASIS)

Dimension	Pubertal Age (years)	r	v	K	$100(1-R_v^2)$	$100(\sum p^2 \div \sum P^2)_v$
Weight	(-2.5)-(+3.5)	0.988	0.9883	0.8630	0.43	—
Height	(-2.5)-(+3.5)	0.981	0.8627	1.858	0.89	—
Sitting height	(-2.5)-(+3.5)	0.976	0.8216	2.064	0.93	—
Trunk length	(-2.5)-(+3.5)	0.983	1.0825	0.6776	1.27	—
Thorax circumf. (nipple)	(-2.5)-(+3.5)	0.982	1.1931	0.3972	0.69	—
Thorax circumf. (xiphoid)	(-2.5)-(+3.5)	0.935	0.9236	1.260	3.37	—
Abdominal circumference	(-2.5)-(+3.5)	0.988	1.0260	0.8347	0.80	—
Total arm length	(-2.5)-(+3.5)	0.961	0.9053	1.305	0.55	—
Arm length	(-2.5)-(+3.5)	0.957	0.9910	0.8774	2.18	—
Hand length	(-2.5)-(+3.5)	0.969	0.7951	1.718	1.53	—
Leg length	(-2.5)-(+3.5)	0.972	0.8767	1.604	1.52	—
Trunk depth	(-2.5)-(+3.5)	0.984	0.8702	1.274	0.83	—
	(-2.5)-(+1.5)	0.956	0.7272	1.895	2.03	73.11
	(+1.5)-(+3.5)	0.983	1.1355	0.5855	1.16	32.78
Neck girth	(-2.5)-(+3.5)	0.964	0.5467	4.192	1.59	—
	(-2.5)-(+0.5)	0.583	0.3274	8.736	12.77	81.41
	(+0.5)-(+3.5)	0.925	0.6383	3.043	0.94	20.09
Bispinous width	(-2.5)-(+3.5)	0.818	1.7435	0.2050	7.91	—
	(-2.5)-(+0.5)	0.724	0.5132	2.611	11.74	22.18
	(+0.5)-(+3.5)	0.495	5.0763	1.5(10 ⁻⁴)	9.47	43.23
Trunk width	(-2.5)-(+3.5)	0.950	0.3934	5.178	1.85	—
Bisacromial width	(-2.5)-(+3.5)	0.964	1.0212	0.9813	1.83	—
	(-2.5)-(+0.5)	0.917	1.6212	0.1361	3.85	42.84
	(+0.5)-(+3.5)	0.921	0.7325	2.636	2.87	56.59
Cephalic length	(-2.5)-(+3.5)	0.796	0.5390	3.664	0.62	—
	(-2.5)-(+0.5)	0.593	1.7874	0.0975	20.92	47.92
	(+0.5)-(+3.5)	0.417	0.2473	8.668	17.76	77.37
Cephalic breadth	(-2.5)-(+3.5)	0.882	0.7738	1.742	6.07	—
	(-2.5)-(+1.0)	0.678	1.1577	0.6267	19.03	85.54
	(+1.0)-(+3.5)	0.957	0.4415	4.263	1.72	19.30

r =product-moment correlation coefficient.

v =sexually differential standardized growth coefficient.

K =sexually differential standardized growth constant.

$(1-R_v^2)$ and $(\sum p^2 \div \sum P^2)_v$ =goodness-of-fit criteria.

A more satisfactory index would be:

Excess growth rate of females over males
Mean growth rate of males and females

$$= 2 \cdot \frac{\left(\frac{dA_f}{A_f} - \frac{dA_m}{A_m} \right)}{\left(\frac{dA_f}{A_f} + \frac{dA_m}{A_m} \right)}$$

$$= 2 \cdot \frac{\left(\left(\frac{dA_f}{A_f} \div \frac{dA_m}{A_m} \right) - 1 \right)}{\left(\left(\frac{dA_f}{A_f} \div \frac{dA_m}{A_m} \right) + 1 \right)}$$

$$= 2 \cdot \frac{l-1}{l+1} \quad (\text{v})$$

Furthermore, if θ is the angular deviation from $\frac{\pi}{4}$ (i.e. from $l=1$) of any particular growth trend,

$$l = \tan \left(\frac{\pi}{4} + \theta \right) = \frac{1 + \tan \theta}{1 - \tan \theta},$$

$$l-1 = \frac{1 + \tan \theta - (1 - \tan \theta)}{1 - \tan \theta} = \frac{2 \tan \theta}{1 - \tan \theta},$$

$$\text{and } l+1 = \frac{1 + \tan \theta + (1 - \tan \theta)}{1 - \tan \theta} = \frac{2}{1 - \tan \theta}.$$

$$\text{Then } \frac{l-1}{l+1} = \left(\frac{2 \tan \theta}{1 - \tan \theta} \right) \div \left(\frac{2}{1 - \tan \theta} \right) = \tan \theta. \quad (\text{vi})$$

For true growth, θ will have a maximum range from $-\frac{\pi}{4}$ through 0 to $+\frac{\pi}{4}$, and $\tan \theta$ will have a range from -1 to $+1$. Such a range is ideal for our purpose, and we have defined *indices of sexual heteroplasia*, denoted by μ , having a range from -1 to $+1$, as:

(1) With reference to a chronological basis,

$$\mu_l = \tan \theta_l = \frac{l-1}{l+1} \quad (\text{vii})$$

and *mutatis mutandis*,

(2) With reference to a developmental background,

$$\mu_v = \tan \theta_v = \frac{v-1}{v+1} \quad (\text{viii})$$

μ_l and μ_v represent in terms of relative growth:

$$\frac{1. \text{ Excess growth rate of female over male}}{2. \text{ Mean growth rate}}$$

Tables IV and V exhibit the values of the indices, in order of increasing heteroplasia, over the whole age-range for the dimensions satisfying the stipulations (1) *a* and (2) *a* above.

As regards satisfaction of condition (3) above, a refinement which could be practised at this stage is

TABLE IV

INDEX OF SEXUAL HETEROPLASIA OVER WHOLE AGE-RANGE FOR SELECTED DIMENSIONS
(CHRONOLOGICAL BASIS)

Rank	Dimension	l	μ_l
1	Trunk length ..	0.9817	-0.0092
2	Weight ..	0.9149	-0.0444
3	Abdomen ..	0.8975	-0.0540
4	Thorax (xiphoid) ..	0.8720	-0.0684
5	Sitting height ..	0.8656	-0.0720
6	Thorax (nipple) ..	1.1623	+0.0751
7	Arm length ..	0.7771	-0.1254
8	Total arm length ..	0.7593	-0.1368
9	Height ..	0.7418	-0.1482
10	Hand length ..	0.7195	-0.1631
11	Leg length ..	0.7030	-0.1744

l = sexually differential growth coefficient.
 μ_l = index of sexual heteroplasia.

the standardization of the indices according to their consistency in conforming to a particular trend, i.e. by weighting with the appropriate values of R^2 . Since the latter is consistently near to 1, this procedure makes no difference to the rank order of the dimensions with regard to heteroplasia (Table VI, overleaf). The dimensions do not differ significantly in their proximity to strictly linear concomitant variation of boys and girls.

Returning to Tables IV and V, we notice firstly that there is a high preponderance of negative indices indicative of heteroplasia in favour of the male, and secondly that there is a considerable difference in the order of heteroplasia on the chronological and developmental assessments. *On a priori* grounds it would be expected that the different methods should give two estimates of the same concept, since much of the net effect of the pubertal acceleration will be common to both sexes, and only the residual

TABLE V

INDEX OF SEXUAL HETEROPLASIA FOR SELECTED DIMENSIONS
(DEVELOPMENTAL BASIS)

Rank	Dimension	v	μ_v
1	Arm length ..	0.9910	-0.0045
2	Weight ..	0.9883	-0.0059
3	Abdomen ..	1.0260	+0.0128
4	Trunk length ..	1.0825	+0.0396
5	Thorax (xiphoid) ..	0.9236	-0.0397
6	Total arm length ..	0.9053	-0.0497
7	Leg length ..	0.8767	-0.0657
8	Height ..	0.8627	-0.0737
9	Thorax (nipple) ..	1.1931	+0.0880
10	Sitting height ..	0.8216	-0.0979
11	Hand length ..	0.7951	-0.1141

v = standardized sexually differential growth coefficient.
 μ_v = index of sexual heteroplasia.

TABLE VI
INDICES OF HETEROPLASIA
STANDARDIZED WITH REFERENCE TO LINEAR CONCOMITANCE

Dimension	Chronological Assessment		Developmental Assessment	
	Rank	$\mu_l \div R_l^2$	Rank	$\mu_v \div R_v^2$
Trunk length ..	1	-0.0094	4	+0.0401
Weight ..	2	-0.0453	2	-0.0059
Abdomen ..	3	-0.0558	3	+0.0129
Thorax (xiphoid) ..	4	-0.0707	5	-0.0411
Sitting height ..	5	-0.0738	10	-0.0988
Thorax (nipple) ..	6	+0.0764	9	+0.0886
Arm length ..	7	-0.1279	1	-0.0046
Total arm length ..	8	-0.1394	6	-0.0500
Height ..	9	-0.1495	8	-0.0744
Hand length ..	10	-0.1670	11	-0.1159
Leg length ..	11	-0.1775	7	-0.0667

μ =indices of sexual heteroplasia.
 R^2 =criteria of linearity.

variation will show itself in the values of μ_l ; and only by virtue of the fact that the pubertal acceleration is approximately equivalent in the two sexes do we satisfy our condition (i.e. linear relationship) with respect to v and μ_v . In spite of the slightly different connotations of l and v consequent upon the exclusion of the younger male and older female age groups in the computation of v , it is legitimate to regard both as representative of the course of relative growth, standardized with regard to differential time of onset of sexual maturation. If we regard μ_l and μ_v merely as different estimates, we are justified in combining them by finding their arithmetic mean. If, on the other hand, we regard

the two sets of indices as not directly comparable but mutually dependent on a common causal system, we can effect a compromise in the order of the dimensions by taking the mean of their rank scores. These two procedures are demonstrated in Table VII.

From Tables IV to VII we can now make our short-list, characterized by close proximity to zero of indices obtained by both appraisals of relative growth between the sexes; weight, trunk length, abdominal circumference, and lower thorax circumference will comprise this short-list. Of these, weight is rejected since it is a cubic measure (see below) and of restricted value for comparison with

TABLE VII
INDICES OF HETEROPLASIA
MEANS AND MEAN RANK SCORES FOR CHRONOLOGICAL AND DEVELOPMENTAL ASSESSMENTS

Dimension	A. $\frac{1}{2}(\mu_l + \mu_v)$		B. Mean Rank Score	
	Rank	Value	Rank	Value
Trunk length ..	1	-0.0152	2	2.5
Abdomen ..	2	-0.0206	3	3.0
Weight ..	3	-0.0252	1	2.0
Thorax (xiphoid) ..	4	-0.0541	5	4.5
Arm length ..	5	-0.0650	4	4.0
Thorax (nipple) ..	6	+0.0816	7=	7.5
Sitting height ..	7	-0.0850	7=	7.5
Total arm length ..	8	-0.0933	6	7.0
Height ..	9	-0.1110	9	8.5
Leg length ..	10	-0.1201	10	9.0
Hand length ..	11	-0.1386	11	10.5

μ_l =index of sexual heteroplasia (chronological basis).

μ_v =index of sexual heteroplasia (developmental basis).

our other dimensions. We must hesitate in giving precedence to circumferential measurements as indicators of the basic growth-potential of the body, since biological considerations lead us to suppose that a number of distinct growth-gradients will be involved. Consequently trunk length is adopted as our standard of reference for estimation of regional differential growth rates.

Additional support for this choice may be obtained from study of both parameters of the allometric equation for trunk length:

$$T_f = 1.074(T_m)^{0.9817}$$

The differential growth constant, as well as the coefficient, is very close to unity, i.e. there is with respect to trunk length a close similarity between the sexes, not only of growth but also of absolute size.

3. SPECIFIC GROWTH COEFFICIENTS

Having selected trunk length on empirical grounds as our sexually invariant dimension, we now proceed to plot, separately for each sex, mean values of any other dimension A for each age group as ordinates on a double logarithmic grid, against the corresponding mean trunk length. Our exploratory graphs fall into two categories according to whether there is evidence of one or two distinguishable linear trends. That the fitted lines are justified as summarizing trends of relative growth during the age periods they represent we may judge by the goodness-of-fit criteria tabulated below.

We may express each phase of growth along a dimension A relative to trunk length (T) as follows:

(a) For males:

$$\log A_m = b \log T_m + h \quad (i)$$

$$\therefore A_m = HT_m^b \quad (ii)$$

Differentiating (i),

$$\frac{dA_m}{dT_m} = b \cdot \frac{A_m}{T_m} \quad (iii)$$

or

$$\frac{dA_m}{A_m} = b \cdot \frac{dT_m}{T_m} \quad (iv)$$

(b) *Mutatis mutandis* for females:

$$\log A_f = g \log T_f + j \quad (v)$$

$$\therefore A_f = JT_f^g \quad (vi)$$

and

$$\frac{dA_f}{dT_f} = g \cdot \frac{A_f}{T_f} \quad (vii)$$

or

$$\frac{dA_f}{A_f} = g \cdot \frac{dT_f}{T_f} \quad (viii)$$

We refer to the parameters b and g , representing growth-partition between the trunk and dimension A , as the specific growth coefficients (S.G.C.s) of the

dimension A for boys and girls respectively. H and J are the specific growth constants. We have, therefore, a convenient method of comparing growth-rates specific for certain dimensions of the body in terms of: (a) proportionate size of the dimension, equations (iii) and (vii); (b) growth rate along trunk length, equations (iv) and (viii).

We employ Huxley's nomenclature to designate change of relative proportions with increase of absolute size as *allometry*. The use of "heterogony" in this sense has been rejected because of ambiguous or alternative meanings (Huxley and Reeve, 1945). Accordingly, we can define dimensions of the body which are growing faster or slower than trunk length respectively as positively or negatively allometric, the appropriate S.G.C.s having values greater or less than unity. For the special category where b (or g) is equal to unity we speak of *isometry*. The existence of *monophasic* or *diphasic* growth patterns for each dimension underlines the theme of Huxley's thesis that specific growth coefficients are normally constant for long periods of the life-history, and from time to time undergo abrupt changes. One dimension (cephalic breadth) appears to be *triphasic* with respect to the female.

The trends of specific growth for all our dimensions are shown by Figs. 3 and 4* (overleaf) and Table VIII (p. 170). For each phase of growth, defined in terms of pubertal age (see Table II), Table VIII exhibits the S.G.C. and growth constant of each dimension in the two sexes, together with the proportionate residual variance, $100(1 - R^2)$, and the product-moment correlation coefficient.

Growth equations *per se* are a fiction and have value only to summarize the average course of growth over a period; but in seeking an understanding of the underlying *Entwicklungsmechanik* at particular stages of the life-history, we can discuss the several levels of allometry estimated for our dimensions during particular phases of growth, taking trunk length as our base-line, i.e. as the definitive isometric axis.†

The specific growth coefficient is a *multiplicative index*, i.e. a ratio of growth rates, and in this form is meaningful; but comparisons referable to allometry of opposite sign introduce a disparity of range, viz. 0 to 1 in the domain of negative allometry and 1 to ∞ in the positive. A derived function of

* For the sake of clarity in comparison of male and female growth, the individual points (paired measurements) have been omitted from these graphs.

† This approach shows up the inadequacy of previous allometric analysis. It is meaningless to speak of allometry unless we state with reference to what dimension the relationship holds. That we need the same basis for comparison is now self-evident; the standard is necessarily arbitrary but should be uniform for the groups compared.

FIG. 3.—Specific growth-rates of selected dimensions, illustrated by logarithmic plotting relative to trunk length (a), (b), and (c) monophasic growth in both sexes; (d) monophasic in females, diphasic in males; (e) diphasic in females, monophasic in males; (f) triphasic in females, monophasic in males.

Broken line: males. Full line: females.

the S.G.C. with a discrete range, ideally of ± 1 about zero, is preferable. Accordingly, we use whichever form of the ratio between the growth coefficients is a fraction of unity: thus if b or $g \leq 1$ we consider

the S.G.C. $\left(= \frac{dA}{A} \div \frac{dT}{T} \right)$ itself; or if b or $g \geq 1$,

we use the reciprocal of the S.G.C. $\left(= \frac{dT}{T} \div \frac{dA}{A} \right)$.

Thus we define an *index of allometry* (ρ) as follows:

(a) if $0 < (\text{S.G.C.}) \leq 1$,

$$\rho_b = b - 1 \text{ and } \rho_g = g - 1$$

(b) if $1 \leq (\text{S.G.C.})$,

$$\rho_b = 1 - \frac{1}{b} \text{ and } \rho_g = 1 - \frac{1}{g}$$

Admittedly, we can provide no *prima facie* justification for this index; but we may judge its usefulness by comparison of two hypothetical

dimensions A and B having S.G.C.s of $\frac{1}{3}$ and

FIG. 4.—Specific growth-rates of selected dimensions, illustrated by logarithmic plotting relative to trunk length. (a), (b), and (c) sexually concordant diphasic growth; (d), (e), and (f) sexually discordant diphasic growth.

Broken line: males. Full line: females.

respectively. Trunk length is then increasing itself at three times the rate of *A*, and *B* is growing three times as fast as trunk length. The equal allometry is shown by the ρ derivatives:

$$\text{for } A, \rho = (\text{S.G.C.} - 1) = -\frac{2}{3}$$

$$\text{for } B, \rho = 1 - \frac{1}{(\text{S.G.C.})} = 1 - \frac{1}{3} = \frac{2}{3}$$

The figures set out in Table IX (p. 171) appear to justify the restricted use of this device, which

preserves the ordinal rank of the S.G.C.s though their absolute magnitudes are not directly comparable, and of course facilitates the comparison of positive and negative coefficients.

4. THE RELATIVITY OF ALLOMETRIC GROWTH

At this stage it is pertinent to comment on dangers inherent in conclusions from growth gradients referable to indiscriminately chosen bodily dimensions. As before, we can write for each

TABLE IX
INDICES OF ALLOMETRY (ρ_b AND ρ_g) FOR SEPARATE PHASES OF SPECIFIC GROWTH

Dimension	BOYS		GIRLS	
	Pubertal Age (years)	ρ_b	Pubertal Age (years)	ρ_g
Weight	(-4.5)-(3.5)	0.6991	(-2.5)-(+5.5)	0.6774
Height	(-4.5)-(+3.5)	0.1625	(-2.5)-(+5.5)	-0.0749
Sitting height	(-4.5)-(+3.5)	-0.0313	(-2.5)-(+5.5)	-0.1405
Thorax circumf. (nipple) ..	(-4.5)-(+3.5)	0.1334	(-2.5)-(+5.5)	0.2647
Thorax circumf. (xiphoid) ..	(-4.5)-(+3.5)	0.0076	(-2.5)-(+5.5)	-0.0989
Abdominal circumference ..	(-4.5)-(+3.5)	0.0028	(-2.5)-(+5.5)	-0.0761
Total arm length	(-4.5)-(+1.0) (+1.0)-(+3.5)	0.2951 -0.1493	(-2.5)-(+1.5) (+1.5)-(+5.5)	0.1987 -0.1894
Arm length	(-4.5)-(+1.0) (+1.0)-(+3.5)	0.3221 -0.2147	(-2.5)-(+1.5) (+1.5)-(+5.5)	0.3007 -0.4309
Hand length	(-4.5)-(+3.5)	0.0916	(-2.5)-(+5.5)	-0.2075
Leg length	(-4.5)-(+1.0) (+1.0)-(+3.5)	0.3095 -0.1314	(-2.5)-(+1.5) (+1.5)-(+5.5)	0.2908 -0.3739
Trunk depth	(-4.5)-(+0.5) (+0.5)-(+3.5)	-0.1995 0.2679	(-2.5)-(+5.5)	-0.0428
Neck girth	(-4.5)-0 0-(3.5)	-0.3672 0.2672	(-2.5)-(+0.5) (+0.5)-(+5.5)	-0.6615 -0.2784
Trunk width	(-4.5)-(+3.5)	0.3303	(-2.5)-(+1.5) (+1.5)-(+5.5)	-0.4525 -0.2442
Bisacromial width	(-4.5)-(+1.0) (+1.0)-(+3.5)	-0.2825 0.2592	(-2.5)-(+1.5) (+1.5)-(+5.5)	0.2582 0.0568
Bispinous width	(-4.5)-(+0.5) (+0.5)-(+3.5)	-0.1597 -0.7779	(-2.5)-(+2.0) (+2.0)-(+5.5)	-0.1082 0.4307
Cephalic length	(-4.5)-0 0-(+3.5)	-0.8049 -0.4172	(-2.5)-(+2.0) (+2.0)-(+5.5)	-0.7304 -0.9755
Cephalic breadth	(-4.5)-(+3.5)	-0.7477	(-2.5)-(+1.0) (+1.0)-(+3.0) (+3.0)-(+5.5)	-0.7286 -0.9416 -0.3579

phase of growth of a dimension A (ordinate)
relative to a second dimension B (abscissa),

and

$$\frac{dA_f}{A_f} = g_{AB} \cdot \frac{dB_f}{B_f} \quad (\text{iv})$$

either $A_m = H_{AB} \cdot B_m^{b_{AB}}$, (i)

or $A_f = J_{AB} \cdot B_f^{g_{AB}}$. (ii)

Thus $\frac{dA_m}{A_m} = b_{AB} \cdot \frac{dB_m}{B_m}$, (iii)

We here designate parameters b_{AB} and g_{AB} ,
being linear gradients referable to the course of
relative (i.e. logarithmic) growth of boys and girls
respectively, as the *relative growth ratio* between
dimensions A and B for males and females. We

TABLE X
RELATIVE GROWTH RATIOS AND CONSTANTS FOR SELECTED PAIRS OF DIMENSIONS

Dimensions $A \times B$	Boys					Girls				
	Pubertal Age	b_{AB}	H_{AB}	$100(1-R^2)$	r	Pubertal Age	g_{AB}	J_{AB}	$100(1-R^2)$	r
Bisacromial width × Neck girth	(-4.5)-(+3.5)	0.9606	1.078	1.54	0.986	(-2.5)-(+0.5)	5.8482	1.48(10 ⁻⁷)	4.50	0.59
	(+0.5)-(+5.5)					(+0.5)-(+5.5)	1.2544	0.4921	2.14	0.95
Bispinous width × Neck girth	(-4.5)-(+0.5)	1.2490	0.1237	8.61	0.836	(-2.5)-(+2.5)	1.5141	0.0549	0.68	0.984
	(+0.5)-(+3.5)	0.2004	4.283	8.82	0.512	(+2.5)-(+5.5)	2.4395	0.0024	1.89	0.926
Cephalic length × Neck girth	(-4.5)-(+3.5)	0.3504	5.653	2.04	0.981	(-2.5)-(+2.0)	0.4880	3.552	4.59	0.86
						(+2.0)-(+5.5)	0.0218	16.82	1.73	0.13
Weight × Height	(-4.5)-(+3.5)	2.7787	0.00107	2.41	0.994	(-4.5)-(+3.5)	3.3499	0.0001	5.23	0.994

b_{AB} =relative growth ratio between dimensions A and B . g_{AB} =relative growth ratio. $100(1-R^2)$ =goodness-of-fit criterion.
 H_{AB} =relative growth constant. J_{AB} =relative growth constant. r =product-moment coefficient of correlation.

use H_{AB} and J_{AB} as corresponding *relative growth constants*. Relevant data for such selected comparisons appear in Table X, in which values of $100(1-R^2)$ justify the use of linear summarizing trends. Fig. 5 (opposite) shows the course of growth along three diphasic dimensions relative to neck girth, which also exhibits a diphasic pattern of growth. Wherever we plot two diphasic dimensions *inter se*, we observe, with reference to the discontinuities, the effect of either *reinforcement* (growth-rates change in opposite directions) or *cancellation* (both increasing or decreasing). The latter may be complete or partial.

On the threshold of puberty in the male, neck girth, bisacromial width, and cephalic length alike enter upon a phase of more intensive growth, while growth of the bispinous dimension decreases. Neck girth and bispinous width of the female, in contrast to bisacromial width and cephalic length, undergo a spurt. Interpretation of Fig. 5 in the light of these is simple against the background of information we derive from choice of an appropriate sexually invariant dimension. It is true that we could justifiably make comparison of growth along any dimensions we choose by always using the same abscissa, e.g. relative to neck girth in Fig. 5a, b, and c; but this would be of limited utility because of the fallibility of comparisons between growth rates of corresponding dimensions in alternate sexes. For instance, we should be tempted to conclude that a dimension A having the same growth-rate with respect to the neck in both sexes was sexually invariant; we should be unable to come to any decision on this question if we were ignorant as to

whether in fact increase of neck girth was itself sexually invariant—a condition for the conclusion stated.

The weight-height relationship is often cited as a parameter descriptive of bodily *habitus*, and the expression $W \propto H^3$ is held to be definitive of this relationship. Table X includes the parameters of the growth equation together with the estimates of conformity to a linear trend. The estimated relative growth ratio of weight to height, 2.78 for males and 3.35 for females, is evidently a sexually differential characteristic. In this connexion a confirmatory technique suggests itself. If we designate the S.G.C.s of dimensions A and B for boys as b_A and b_B respectively,

$$\text{then } \frac{dA_m}{A_m} = b_A \cdot \frac{dT_m}{T_m},$$

$$\text{and } \frac{dB_m}{B_m} = b_B \cdot \frac{dT_m}{T_m},$$

so that, other things being equal,

$$\frac{dA_m}{A_m} = \frac{b_A}{b_B} \cdot \frac{dB_m}{B_m} \\ \therefore \frac{b_A}{b_B} = b_{AB} \quad (v)$$

The qualification is necessary since the argument assumes that the residual variation about the linear trends for A and B is negligible and that there is no special conformity or retro-action between A and B which would not show in estimations of growth rates relative to trunk length. We shall obtain

FIG. 5.—Indiscriminate selection of a standard dimension for comparing growth-rates in the two sexes. (a)-(c), showing use of neck girth as a yardstick, give pictures which are *prima facie* misleading (cf. Fig. 4). (c) and (d) contrast patterns of growth of bispinous width relative to different standards.

Broken line: males. Full line: females.

approximate agreement with the formal postulate of equation (v) where there is close correlation of growth processes of the dimensions A and B and trunk length, and more divergence where the summarizing trends of which b_A , b_B , and b_{AB} are parameters are less exact, although each may be valid in its own right. The S.G.C.s for height and weight and the ratio of these, with the relative growth ratio for these dimensions *inter se* have been

tabulated (Table XI, overleaf), and it will be seen that there is a strikingly close agreement, which enhances the validity of the method we are exploiting.

Application of the same procedure to other pairs of dimensions shows that the approximations and assumptions in the confirmatory technique are not always justified (Table XI). Furthermore, the age-delimitations of the discrete phases of growth do not always conform for the two regions in question,

TABLE XI
COMPARISON OF RATIO OF S.G.C.S WITH RELATIVE GROWTH RATIO

$A \times B$	Sex	Phase	(S.G.C.) _A	(S.G.C.) _B	$\frac{(S.G.C.)_A}{(S.G.C.)_B}$	Relative Growth Ratio
Weight × Height	M.	(monophasic)	3.3240	1.1936	2.7849	2.7787
	F.	(monophasic)	3.0928	0.9251	3.3432	3.3499
Neck girth × Weight	M.	1	0.6328	3.3240	0.1904	0.2352
		2	1.3646		0.4107	0.3432
	F.	1	0.3385	3.0928	0.1094	0.1000
		2	0.7216		0.2333	0.2351
Bispinous width × Neck girth	M.	1	0.8403	0.6328	1.3279	1.2490
		2	0.2221	1.3646	0.1628	0.2004
	F.	1	0.8918	0.3385	2.6346	1.5141
		2	1.7571	0.7216	2.4350	2.4395
Cephalic length × Bispinous width	M.	1	0.1951	0.8403	0.2321	0.1678
		2	0.5828	0.2221	2.6240	1.4357
	F.	1	0.2696	0.8918	0.3024	0.2903
		2	0.0245	1.7571	0.0066	0.0023

so that in effect we may be comparing the ratio of the S.G.C. of dimension A over a period of x years to that of B over y years, with the relative growth ratio of A to B over z years.

An alternative confirmatory technique is a corollary to equation (v). If the S.G.C.s for a dimension A , for the whole age-period, for boys and girls respectively are b_A and g_A ,

so that

$$\frac{dA_m}{dT_m} = b_A$$

and

$$\frac{dA_f}{dT_f} = g_A$$

whence

$$\frac{g_A}{b_A} = \frac{dA_f}{dA_m} \cdot \frac{dT_m}{dT_f}$$

TABLE XII
COMPARISON OF RATIO OF S.G.C.S WITH SEXUALLY DIFFERENTIAL GROWTH COEFFICIENT

Dimension A	S.G.C.			$\frac{g_A}{b_A} \cdot l_T$	l_A
	b_A	g_A	$\frac{g_A}{b_A}$		
Weight	3.3240	3.0928	0.9304	0.9134	0.9149
Height	1.1936	0.9251	0.7751	0.7609	0.7418
Sitting height	0.9687	0.8595	0.8873	0.8711	0.8656
Thorax (xiphoid)	1.0081	0.9011	0.8939	0.8775	0.8720
Thorax (nipple)	1.1537	1.3599	1.1787	1.1571	1.1623
Abdominal circumference	1.0033	0.9239	0.9209	0.9040	0.8975
Hand length	1.1007	0.7925	0.7200	0.7068	0.7195

b_A = S.G.C. (male) of dimension A .

g_A = S.G.C. (female) of dimension A .

l_A = sexually differential growth coefficient of dimension A .

l_T = sexually differential growth coefficient of trunk length, (≤ 1).

and if the slope of the *mean* trend of growth of girls relative to boys along A , again for the complete age-range studied, is l_A (see Table I)

so that
$$\frac{dA_f}{dA_m} = l_A,$$

then
$$l_A = \frac{g_A}{b_A} \cdot \frac{dT_f}{dT_m},$$

i.e.
$$l_A = \frac{b_A}{g_A} \cdot l_T; \quad (\text{vi})$$

and as $l_T = 1$, according to our assumptions,

$$l_A = \frac{g_A}{b_A}. \quad (\text{vii})$$

Since S.G.C.s and sexually differential growth coefficients are estimates, we introduce a new source of error by computing the ratio of two such indices. Nevertheless, the procedure reinforces our confidence in the use of the allometric equation. Table XII exhibits the sexually differential growth coefficients, and analogous approximations derived from specific growth coefficients, for dimensions showing a constant S.G.C. throughout our age-group. There is very close agreement between $\frac{g_A}{b_A} \cdot l_T$ and l_A .

When we give l_T its theoretical value ($=1$), the correspondence between $\frac{g_A}{b_A}$ and l_A is also good.

Such correspondence justifies this type of analysis and vindicates the choice of trunk length as a basis for comparison.

5. SEX DIFFERENTIALS IN SIZE AND SHAPE

The term *sexual heteroplasia* has already been introduced to denote differential growth rates between the sexes, and *sexual dimorphism* here signifies, in its literal sense, differential size and shape. The two concepts are, of course, interdependent; sexual dimorphism is a direct result of heteroplasia, but the same organ may at different stages in the life cycle be: (a) sexually isomorphic and heteroplastic; (b) sexually dimorphic and homoplastic. The condition for dimorphism is a state of heteroplasia over some period of the life history; but successive phases of positive and negative heteroplasia, or *vice versa*, in the same dimension may result in isomorphism.

Differential and specific growth coefficients are in fact growth ratios standardized with respect to the differential size of the dimensions compared, and offer no direct indication of the latter. We can obtain an estimate of relative size from the general formula $A_f = CA_m^l$, viz. $\frac{A_f}{A_m} = CA_m^{l-1}$, but it is simpler and in addition useful as a confirmatory device to compute such directly from the data. We can investigate sexual dimorphism between boys and girls of the same chronological age, and thus observe

TABLE XIII
INDEX OF SEXUAL DIMORPHISM
(CHRONOLOGICAL BASIS)

Dimension	s for Age (years)			
	8.75	11.25	13.25	16.75
1. Weight	0.0217	-0.0164	0.0615	-0.0586
2. Height	0.0167	-0.0045	0.0168	-0.0238
3. Sitting height	0.0015	-0.0045	0.0021	-0.0250
4. Trunk length	0.0011	-0.0117	0.0152	-0.0061
5. Abdominal circumference	0.0159	-0.0147	0.0020	-0.0171
6. Thorax circumference (nipple)	-0.0106	-0.0153	0.0238	-0.0087
7. Trunk depth	-0.0215	-0.0439	-0.0009	-0.0890
8. Neck girth	-0.0133	-0.0282	-0.0130	-0.0732
9. Arm length	-0.0105	-0.0415	-0.0212	-0.0637
10. Total arm length	-0.0032	-0.0256	-0.0090	-0.0549
11. Cephalic length	-0.0147	-0.0170	-0.0050	-0.0462
12. Leg length	0.0224	0.0033	0.0207	-0.0309
13. Trunk width	-0.0068	-0.0205	-0.0464	-0.1130
14. Thorax circumference (xiphoid)	-0.0272	-0.0136	0.0045	-0.0425
15. Bisacromial width	0.0195	0.0490	0.0737	0.0382
16. Bispinous width	-0.0020	-0.0038	-0.0077	0.0454
17. Hand length	0.0111	0.0178	0.0250	-0.2977
18. Cephalic breadth	-0.0231	-0.0151	-0.0131	-0.0210
19. Bizygomatic width	0.0176	0.0245	0.0276	0.0227

N.B.—Positive values indicate larger size of female.

changes in its nature and extent *pari passu* with constant or changing phases of heteroplasia. To do this we have constructed an *index of sexual dimorphism* (s) analogous to the *indices of heteroplasia* (see equations (vi) to (viii)). For a dimension A , with the usual notation,

$$s = \frac{1}{2} \cdot \frac{\text{Excess size of female dimensions over male}}{\text{Mean size of dimension in the two sexes}}$$

$$\text{i.e.} \quad s = \frac{A_f - A_m}{A_f + A_m}.$$

It is not possible to construct an index which has a convenient range and also retains a precise significance in terms of relative size. The index s has in practice a limited range, and a value of zero signifies isomorphism. So defined, *positive* dimorphism signifies larger size of the female in the relevant dimension, *negative* dimorphism the reverse.

We have seen that we may distinguish three phases of differential growth in the age range studied: (i) both sexes are prepubescent; (ii) girls only have entered upon puberty; (iii) the process has already started in both sexes. Table XIII exhibits values of s for the youngest chronological age-group in the sample, for the estimated ages of onset of each succeeding new phase of relative growth, and for the upper age limit of our records. Examination of this table reveals what we might expect:

- (a) The index s is lower at the beginning of puberty in females than at the start of the age range, i.e. girls are in general smaller relative to boys at 11.25 years than at 8.75. Indeed, at the time of onset of puberty in girls, their size is absolutely smaller than boys of the same age along all dimensions studied except shoulder width, hand length, and leg length.
- (b) When girls have been pubescent for two years and boys are on the verge of the process, they are bigger than boys along most dimensions, and where still absolutely smaller, are less so than they were two years previously, the only exception being referable to trunk width.
- (c) For our oldest age group there is again a preponderance of negative indices; boys are bigger than girls at this age along all dimensions except bisacromial and bispinous widths.

Computation of the proportionate size of pairs of dimensions at particular ages (Table XIV, opposite) gives an immediate clue to the somatic outline characteristic of each sex. Trunk length contributed slightly more to total stature in boys than in girls in the youngest age group and less in the oldest age group. This is *en rapport* with: (a) a smaller proportionate contribution of leg length in young boys than in girls of the same age group; (b) the reversal

of the situation for the older ages; (c) what our figures disclose concerning growth of legs relative to trunk. In short, there is greater emphasis on trunk in the contour of the prepubescent boy and on legs in that of the prepubescent girl. During the course of puberty, the picture changes. Boys become proportionately longer in the leg than girls. Arms of boys are throughout proportionately longer than those of girls; and the female arm is appreciably shorter in relation to the leg than is that of the male. Trunk width becomes proportionately larger in boys and smaller in girls, and trunk depth is consistently greater in boys. Bisacromial width becomes proportionately greater in girls and less in boys; but there are indications which point to a reversal of these trends subsequent to the age of our oldest group. Bispinous width of boys becomes smaller in relation to trunk length while that of girls remains fairly constant.

6. MORPHOGENETIC PATTERNS OF SEXUAL MATURATION

If we attribute the inception of a new phase of growth at puberty to a specific stimulus incident to the sexual metamorphosis, we can conveniently subsume our dimensions within the following fourfold taxonomy:

- Type 1. Growth monophasic in both sexes.
- Type 2. Growth monophasic in males, diphasic (or triphasic) in females.
- Type 3. Growth monophasic in females, diphasic in males.
- Type 4. A. Sexually concordant diphasic growth.
B. Sexually discordant diphasic growth.

Our main findings for each dimension studied, thus classified, appear in Table XV (overleaf). In almost every case, our criteria of reliability vindicate what the figures convey. Against this background we are in a position to present a more coherent picture of sexually differentiated growth. To avoid periphrasis, it is necessary to emphasize that all statements which follow refer to group averages in contradistinction to individuals.

Trunk.—The depth of the trunk does not greatly alter proportionally in either sex. Younger boys (age 8.75) are isomorphic with girls in length, and are both thicker and broader (at all levels measured). With increasing age, length remains uniform, boys continue to be thicker than girls and become increasingly broader. Trunk width forms an increasingly greater proportion of thorax circumference at the nipple in boys (Table XIV), a fact which is responsible for the unexpectedly high rate of increase of the latter dimension in boys and its

TABLE XIV
PROPORTIONATE SIZE OF SELECTED DIMENSIONS

Dimensions	Boys				Girls			
	Ratio for Age (years)				Ratio for Age (years)			
	8·75	11·25	13·25	16·75	8·75	11·25	13·25	16·75
$\frac{\text{Sitting height}}{\text{Height}}$	0·5531	0·5269	0·5246	0·5218	0·5363	0·5269	0·5290	0·5205
$\frac{\text{Trunk length}}{\text{Height}}$	0·3279	0·3214	0·3223	0·3131	0·3178	0·3168	0·3213	0·3244
$\frac{\text{Leg length}}{\text{Height}}$	0·5246	0·5361	0·5409	0·5518	0·5306	0·5445	0·5452	0·5448
$\frac{\text{Leg length}}{\text{Trunk length}}$	1·6000	1·6680	1·6781	1·7622	1·6695	1·7189	1·6970	1·6770
$\frac{\text{Total arm length}}{\text{Trunk length}}$	1·3576	1·3959	1·4153	1·4736	1·3458	1·3578	1·3487	1·3366
$\frac{\text{Hand length}}{\text{Total arm length}}$	0·2742	0·2575	0·2559	0·2530	0·2846	0·2808	0·2739	0·2662
$\frac{\text{Leg length}}{\text{Total arm length}}$	1·1786	1·1949	1·1857	1·1958	1·2405	1·2660	1·2582	1·2547
$\frac{\text{Trunk width}}{\text{Trunk length}}$	0·3990	0·3915	0·4092	0·4553	0·3928	0·3847	0·3618	0·3673
$\frac{\text{Trunk depth}}{\text{Trunk length}}$	0·3483	0·3475	0·3396	0·3718	0·3329	0·3259	0·3288	0·3304
$\frac{\text{Trunk width}}{\text{Thorax circumf. (nipple)}}$	0·2586	0·2701	0·2685	0·2803	0·2606	0·2556	0·2333	0·2273
$\frac{\text{Bisacromial width}}{\text{Trunk length}}$	0·6263	0·5902	0·5803	0·6097	0·6498	0·6664	0·6525	0·6662
$\frac{\text{Bispinous width}}{\text{Trunk length}}$	0·1802	0·1748	0·1763	0·1652	0·1806	0·1776	0·1737	0·1832
$\frac{\text{Bispinous width}}{\text{Bisacromial width}}$	0·2878	0·2963	0·3039	0·2710	0·2779	0·2666	0·2662	0·2750

isomorphism in spite of the effect of inclusion of the female breasts. Trunk width also progressively increases in boys in relation to trunk length, and decreases in girls. Trunk depth is greater in proportion to length in boys and tends to become

increasingly so. Vertically and horizontally in the dorsoventral plane, there is a sharp spurt of growth at the beginning of puberty in either sex; but trunk width appears to respond to a specifically male stimulus.

TABLE XV
BALANCE-SHEET OF PUBESCENT GROWTH

Dimension	Sexual Heteroplasia			Sexual Dimorphism			Allometry					
	Phase			8·75-16·75 years	8·75 years	16·75 years	Female			Male		
	Prepubescent	♀ pubescent ♂ prepubescent	Pubescent				Phase 1	Phase 2	Pubertal age at inception of new phase	Phase 1	Phase 2	Pubertal age at inception of new phase
Trunk length ..	—	++	--	0	0	0	0	...	...	0	0	...
Monophasic in both sexes												
Weight ..	--	++	--	0	+	--	+++	...	...	+++	...	...
Height ..	—	++	--	0	0	—	0	...	...	0	...	...
Sitting height ..	—	++	--	0	0	—	—	...	...	+	...	...
Thorax circ. (nipple)	0	++	--	0	0	0	0	...	...	0	...	...
Thorax circ. (xiphoid)	0	++	--	0	—	—	0	...	...	0	...	...
Abdominal circ.	--	++	--	0	+	—	0	...	...	0	...	...
Hand length ..	0	+	--	—	+	—	—	...	...	0	...	...
Monophasic in males												
Polyphasic in females												
Trunk width ..	--	--	--	0	0	---	---	—	+1·5	+	—	...
Cephalic breadth ..	0	0	0	0	—	---	(a) --- (b) ---	---	+1·0 +3·0	...	...	...
Monophasic in females												
Diphasic in males												
Trunk depth ..	--	++	--	0	—	---	0	...	...	—	+	+0·5
Diphasic in both sexes												
Sexually concordant												
Total arm length ..	—	+	--	—	0	---	+	—	+1·5	+	—	+1·0
Arm length ..	--	+	--	—	0	---	+	---	+1·5	+	—	+1·0
Leg length ..	—	+	--	—	+	---	+	---	+1·5	—	—	+1·0
Neck girth ..	--	+	--	—	—	---	—	---	+0·5	—	+	0
Sexually discordant												
Bisacromial width ..	++	++	--	0	+	++	+	0	+1·5	—	+	+1·0
Bispinous width ..	—	++	++++	++	0	---	—	++	+2·0	—	---	+0·5
Cephalic length ..	+	+	---	—	—	---	---	---	+2·0	---	---	0

It is unfortunate that measurements are lacking for trunk width and depth at the level of the umbilicus. Such data might well reveal an underlying sex difference in body shape as at nipple level. In this context, it is also appropriate to call attention to the partition of total stature between head-plus-neck, trunk, and legs. Table XIV shows that trunk length contributes rather less than one-third to total height, and that the ratio tends to decrease slightly in boys and to increase in girls. Throughout our age-range, sitting height is a little over half of total stature of boys and girls alike. The ratio is greater for boys than for girls of the youngest age group; but the two differ little in the oldest group. Proportionately, the leg of a girl is somewhat longer than that of a boy at all ages. It contributes just over 50 per cent. to total height and increases relative to height in both sexes. Both trunk length and leg length of girls increase in proportion to total stature while the proportionate contribution of sitting height decreases. We may infer, therefore,

that the contribution of head-plus-neck to total height undergoes a slight progressive decrease in the female. Leg length of boys becomes greater relative to height, while sitting height and trunk length decrease their contribution to total stature. Seemingly, therefore, head-plus-neck height does not change appreciably.

Limbs.—Growth of arm length (to wrist) and leg length (to ankle) alike accelerate at the beginning of puberty (*c.* 11·25 and 13·25 years in girls and boys respectively), i.e. at the same time as, and in the same proportion to, acceleration of trunk length. There is subsequently a decrease of relative growth. Thus the limbs are synchronized in showing an early falling-off of growth, with early attainment of their size limit. Less certainly, the hand shows a similar tendency for early decline of relative growth. Since the hand of younger girls is longer than that of boys of the same age, total arm length is isomorphic for the group aged 8½-9 years. Subsequently, the male hand grows more rapidly than the female, and total

arm length is greater in boys for our oldest group. The leg of the female is considerably longer in relation to the arm (including hand) than is that of the male (Table XIV) throughout our age-range, whilst both increase more rapidly in relation to trunk length in boys.

Arm length has an S.G.C. of 1.47 in boys, subsequently falling to 0.79 some time after the onset of puberty, the corresponding figures for girls being 1.43 and 0.57. The S.G.C. of hand length, concerning which there was insufficient evidence for the putative second phase of diminished growth, is 1.10 for boys and 0.79 for girls, a lower net rate of relative growth than that of arm length. The allometric approach thus offers substantial evidence to support the postulate of a proximal-distal gradient of growth-potential. This may signify that the more distal part of the organ grows more slowly than the proximal region, or that it is more refractory to stimuli.

Head.—The scatter of each of our three cephalic measurements is greater than for most other dimensions. From what has been said in Section 3, we should expect cephalic length to become greater relative to cephalic breadth in boys and the converse to be true of girls. Fig. 6 depicts the trends and Table XVI presents the numerical data. There is for boys a slight decrease and for girls a lower rate of increase of the cephalic index (C.I.) relative both to age and to trunk length. By plotting the C.I. for girls

TABLE XVI
CEPHALIC INDEX
TRENDS IN RELATION TO AGE AND TRUNK LENGTH

	Slope	
	Boys	Girls
C.I. $\times$ Age	-0.1516	+0.0257
C.I. $\times$ Trunk length	-0.0853	+0.0144

against that for boys on a logarithmic scale, it is impossible to discern any concurrence between the sexes. Our results substantiate the general conclusion that the mean C.I. increases in a female population and decreases in a male population, and that the two trends cross during puberty at the estimated age of 12 years. In other words, boys seem to be more dolichocephalic than girls before the age of 12, and after 12 more brachycephalic; but the variability which our observations disclose, whether attributable to insufficient refinement of measuring technique or inherent in the population (or, as is more likely, ascribable to both), compels us to state this conclusion with caution.

Fig. 7 (overleaf) is a calendrical visualization of our estimation of the time relations of the main features of the sexual metamorphosis, including those investigated in Part I of the present studies. Dimensions specifically exhibiting a spurt or deceleration relative to trunk length have been distinguished by asterisks. At the upper limit of our sample, 17 years, growth is continuing in both sexes, presumably at decreasing absolute rates. As growth slows down, allometric analysis may become increasingly unreliable as the relative fluctuation of measurements tends to increase.

We define the onset of puberty arbitrarily as the age at which the first qualitative indication appears. We have seen that this coincides with a spurt of growth along certain dimensions, and there may well be other and earlier signs of pubescence outside the scope of our observations. Our method of estimating the age at which growth accelerates is subject to an error of about six months, being dependent on the localization of discontinuities in a diphasic or triphasic pattern of growth. With due regard to this *caveat*, salient features emerging from a study of time relations are: (a) that girls anticipate boys by 2-2½ years in the onset of sexual maturation; (b) that in both sexes successive manifestations of puberty may overlap; (c) that with respect to allometric growth boys appear to undergo a more intense process of shorter total duration.

FIG. 6.—Age-group means and summarizing trends for Cephalic Index.

FIG. 7.—Summary of the main qualitative and quantitative manifestations of pubescence.

The previous communication (Hogben and others, 1948) focused attention on sex-specificity of pubescent growth of the bispinous and bisacromial dimensions and neck girth. Provisional conclusions there advanced were: (i) that a male hormone is decisive for neck growth; (ii) that a female hormone is decisive for accelerated growth of the pelvis; (iii) that the male and female gonads may exercise a contrary influence on bisacromial width, possibly indicative of the existence of a male growth-promoting and a female growth-retarding hormone.

If sex hormones alone are indeed responsible, directly or indirectly, for the secondary characteristics of the sexual metamorphosis, we have at our disposal several possible hypotheses, of which it will suffice to specify three: (a) the secretion of a different hormone by each sex, the two having some effects in common, others opposite; (b) the production of two hormones by each sex, one sex-specific and one common to males and females; (c) a quantitative difference between the relative concentrations of two hormones, each present in each sex. At the same time we must allow for the possibility, and indeed the likelihood, of special, and sexually differential, threshold responses of specific regions or organs to one and the same hormone. It is somewhat unlikely that the female pelvis would be stimulated to increased growth, and the male pelvis to retarded growth, as a result of the activity of the

same hormone in the same, or even in different concentrations. Similar considerations apply to shoulder width and cephalic length.

Whilst we can come to no definite conclusion, the most attractive hypothesis we might formulate, if exclusively to explain our own results, is (b) above. The widely-held, but not satisfactorily established, conception of the presence of closely related groups of androgens and oestrogens in different relative intensities in the two sexes is *en rapport* with, but not a necessary condition for, our findings. Androgens are commonly held to have a growth-stimulating and trichogenic effect, and may be associated with our postulated common factor, whilst oestrogens have effects specific to the female. A more satisfactory possibility is that the presence of the androgen group of hormones is associated with the features of puberty common to both sexes and specific to the male, and that oestrogens are largely responsible for specifically female characteristics. This hypothesis is supported circumstantially by our analysis, as well as by evidence both from experimental physiology and from endocrine pathology; nor should we forget that production of androgen-like substances as well as oestrogens by the ovary has been established more conclusively than the existence of both groups in the male. Also the more gradual onset of the growth spurt in the female may be due to a lower concentration of androgens, either

relatively or absolutely, than in the male. Until a critical appraisal of the literature relating to experimental physiological and pathological approaches to the study of sex hormones has been made, and augmented by crucial experiment, it will not be possible to formulate a more satisfactory explanation of causal mechanisms operative during puberty. It is not even possible to assert with confidence that the accelerated growth of the body at puberty is a direct result of gonadal secretions; increased secretion of the anterior lobe pituitary growth hormone may be the immediate stimulant of pubertal growth. It is, however, convenient, within the limits of the information at present at our disposal, to regard the events shown in Fig. 7 as consequent upon the onset of the secretory activity of the ovary and the testis.

We have demonstrated that the event usually taken as the signal of puberty in females, the onset of menstruation, does not in fact occur until over two years after other diacritical events. It appears that the cyclical activity of the ovary is a threshold effect of hormone concentrations, and this is supported by the frequent irregularities of the menstrual cycle during adolescence. Definition of the onset of puberty by menarche, though etymologically permissible, is physiologically misleading.

It is outside the range of the present communication to discuss relative somatic proportions of males and females in the adult state; but an interesting possibility suggests itself to explain the larger size of the adult male, already apparent in our oldest age group. At the age of 9, girls exceed boys in height, weight, and several other dimensions representative of body size in general, and at the beginning of puberty there is in both sexes a spurt in growth of the body as a whole. According to the bulk of published evidence, absolute growth follows an approximately logarithmic curve. In any case it slows down after acceleration at puberty, and eventually comes to a standstill approximately 10 years later. Since boys start puberty two years after girls, we might expect them to stop growing later. Other things being equal, men might well therefore be, as in fact they are, larger than women in virtue of two additional years of prepubertal growth; but possibly also contributory to the greater size of the human male is a slightly greater intensity both of prepubertal growth and of the pubertal spurt.

CONCLUSIONS

1. The allometric equation $y=kx^b$ is satisfactory as an empirical method of analysis of the morphogenetic pattern of human prepubertal and pubertal development.

2. Special attention should be focused on the method of curve-fitting in the application of this and other growth equations. A method of fitting (least square-perpendiculars) has been found more suitable and is also satisfactory *vis à vis* goodness-of-fit criteria.

3. Females start puberty 2-2½ years earlier than males; the inception is marked by the start of the enlargement of the mammary glands in the female and by the appearance of pubic hair in the male, concurrently with the onset of a tempo of more intense growth along nearly all dimensions in both sexes.

4. Trunk length is virtually a sexually invariant dimension with reference to rate of growth and absolute size, and is therefore *de facto* a suitable basis for the comparison of specific growth rates of other dimensions in the same and opposite sexes. Accordingly, we speak of the *specific growth rate* of a particular dimension as its growth relative to that of the trunk.

5. During puberty, there is no noteworthy alteration for either sex of the tempo of growth of the following dimensions relative to trunk length:

- Weight
- Height
- Sitting height
- Thorax circumference (at nipple and xiphoid process)
- Abdominal circumference
- Hand length

6. Change of the specific growth rate at some time during puberty is characteristic of certain dimensions as indicated below:

(a) in females alone:

- Trunk width (acceleration)
- Cephalic breadth (retardation one year after, and acceleration three years after, inception of puberty)

(b) in males alone:

- Trunk depth (acceleration)

(c) in both sexes:

- Arm length, with and without hand (retardation)
- Leg length (retardation)
- Neck girth (acceleration, more intense in males)
- Bisacromial width (acceleration in male, retardation in female)
- Bispinous width (retardation in male, acceleration in female)
- Cephalic length (acceleration in male, cessation of growth in female)

7. Limb growth (arm and leg) undergoes a spurt at the onset of puberty and declines thereafter more steeply than growth of the trunk.

8. The following dimensions exhibit a well-defined heteroplasia in our age range:

(a) *Positively heteroplastic* (female grows faster):

Bisacromial width (prepubescent* only)

Bispinous width (postpubescent† only)

(b) *Negatively heteroplastic* (male grows faster):

Hand length

Trunk width

Arm length (with and without hand)

Leg length

Neck girth

Bisacromial width (postpubescent)

Bispinous width (prepubescent)

Cephalic length (postpubescent)

9. The following dimensions are characterized by sexual dimorphism in childhood (at 8·75 years):

(a) *Positive* (larger size in female):

Weight

Height

Abdominal circumference

Leg length

Bisacromial width

(b) *Negative*:

Trunk depth

Neck girth

Cephalic length

Thorax circumference (xiphoid process)

Cephalic breadth

10. The following dimensions are significantly dimorphic in adolescence (at 16·75 years):

(a) *Positive*:

Bisacromial width (dimorphism decreasing)

Bispinous width (dimorphism increasing)

(b) *Negative*:

Weight

Height

Sitting height

Abdominal circumference

Leg length

Trunk length

Trunk depth

Neck girth

Arm length (with and without hand)

Hand length

Thorax circumference (xiphoid process)

Cephalic length

Cephalic breadth

11. From these conclusions it is possible to derive a composite picture of differential body-shape with reference to sex: (a) males become thicker and broader in the trunk, and longer in the leg, than females; (b) thoracic circumference becomes slightly greater in males, because the increased trunk depth

and width compensates for the enlargement of the breasts in the female.

12. Cephalic measurements (length, breadth, and bizygomatic width) show more variability between mean values for successive age-groups, especially in males, than is characteristic of other dimensions. Bizygomatic width is greater in girls but shows too much fluctuation to be amenable to satisfactory estimation of relative growth. Over the whole range of 8½-17 years, the cephalic index shows a slight increase in girls, but at 12 years there is no apparent sex difference. In so far as our sample is representative, we may therefore say that before the age of 12 a female population is on the average more dolichocephalic than a male population, and that after 12 it is more brachycephalic.

13. It is often stated that the relationship between weight and height can be expressed in the form:

$$W \propto H^3$$

A more exact relationship appears to hold good as a sexually differential characteristic within our age-range:

$$\text{males: } W \propto H^{2.78}$$

$$\text{females: } W \propto H^{3.35}$$

The growth ratio of weight to trunk length is more sexually invariant:

$$\text{males: } W \propto T^{3.32}$$

$$\text{females: } W \propto T^{3.09}$$

14. At the age of 17, boys are larger than girls with respect to all the main dimensions of the body, except pelvic width, shoulder width, and bizygomatic width. It is suggested that a contributory cause of the larger size of the human adult male is the delayed release of the sex hormone (or hormones), with a subsequent delay of two years in the final decline of body growth, i.e. the female undergoes two years' prepubescent growth less than the male.

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* before the inception of puberty.

† after the inception of puberty.