BOOK REVIEW

On the Objective Study of Crowd Behaviour. By L. S. PENROSE. Pp. 74. H. K. Lewis, London. 1952. (10s.)

In this brochure Professor Penrose puts forward one novel, challenging, and highly original idea buried in a banal matrix of tedious metaphor and metalepsis about the behaviour of men in groups and their reactions to micro-organisms and viruses. Those who fail to derive any profit from the analogy between crowd diseases as Greenwood uses the term and crowd disorders as Penrose does may also miss the point which makes the publication of the essay more than worth while. True to the Galton Laboratory tradition, the author assumes that the reader, if also a mathematician, will immediately grasp the statistical theory he advances; and, if not, will be too dumb to do so. This is a pity, because a public of thoughtful people is getting more and more suspicious of statistical generalizations advanced for allegedly adequate theoretical reasons when there is, as for the so-called cube law, merely a somewhat exiguous empirical basis to support them.

The Penrose square-root law has also to do with voting; and what follows is an attempt to fill in the argument which the author himself does not deign to elaborate. The elaboration is all the more pertinent because his hope that the reader "will tolerate the necessary introduction of mathematical notation" (p. 6) immediately precedes three gross errors in the formulae which follow, *viz.*:

$K/\sqrt{n_r}$ on p. 7 should presumably be $K\sqrt{n_r}$,

$n_R=1/n_I$ should presumably be $n_R=n_I$,

the differential element in the definite integral on the same page is missing; and $\sqrt{2}\pi$ in this formula should be $\sqrt{2\pi}$.

Fortunately, the Table at the foot of p. 7 permits one to reconstruct the line of thought from which the author's pen has erred so consistently. One may state it as follows: If a collection of n indifferent voters registers choice at random on a yes-no issue, the distribution of ayes or the reverse tallies with the terms of the binomial $(\frac{1}{2} + \frac{1}{2})^n$. If we denote the number of ayes by the score x , its deviation from the mean is $(x - \frac{1}{2}n) = X = \frac{1}{2}(2x - n)$. The excess of ayes (*majority*, if positive) is $x - (n - x) = (2x - n)$, so that $2X = m$, if positive, is the majority for the affirmative. The variance of the score distribution is $\frac{1}{4}n$ so that the square standard score corresponding to x is

$$c^2 = \frac{X^2}{\frac{1}{4}n} = \frac{m^2}{n} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (i)$$

If n is fairly large (*e.g.* 20 or over) we may treat this as a χ^2 variate whose mean value is unity. Thus the mean value of m^2 should be n , if the n voters acted independently at random. If observation of many elections in presumptively stable conditions shews that the mean value of m^2 for an observed mean value (n_m) of n is in fact $k^{-1}.n_m$, the voting system operates as if $k^{-1}.n_m$ individuals replace n_m , or as if k individuals on the average behave as a block.

This aspect of the Penrose square-root law implicit in equation (i) is of secondary importance to the way in which a small block acting as one individual can control most decisions. In addition to n indifferent voters we postulate in the derivation of (i), we may also

postulate a compact block of b individuals who will always vote against the *ayes*, so that the *ayes* will not get a majority unless $m > b$; *i.e.*

$$c > \frac{b}{\sqrt{n}} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (ii)$$

With the value of c so defined, the probability that the *ayes* will *not* get a majority is then, of course

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^c e^{-\frac{1}{2}c^2} . dc$$

This defines 97.7 per cent. of the area of the normal curve when $c = 2$, as when $b = 200$ and $n = 10,000$, or when $b = 2,000$ and $n = 1,000,000$. Examination of voting records in the light of considerations advanced earlier strongly suggests that communities such as Great Britain and the U.S.A. behave at the polls as random blocks. In a community of 50,000,000 containing 25 such blocks of 2,000,000 (and Penrose puts as a rough and ready figure about 15 for U.K.), one disciplined group of 2,000,000 could control about 85 per cent. of all decisions.

Professor Penrose would be first to admit that such considerations are schematic, but they are not on that account profitless. They bring a new searchlight to bear on the limiting size of an organization (political, industrial, or professional) consistent with effective two-way traffic of ideas, and may contribute to a sane revaluation of alleged benefits from organization on a large scale. Professor Penrose commendably draws attention to an aspect of the large-scale community which is of special constructive interest if civilized humanity is to survive the challenge of the atomic era. For the essay concludes with a table of representation of nationals in a world assembly calculated on the basis of the square-root law to ensure minimal dangers of undue influence by small pressure groups. LANCELOT HOGBEN.