BOOK REVIEW

Probability and the Weighing of Evidence. By I. J. GOOD, M.A., Ph.D. Charles Griffin, London. Pp. 119 + viii. (16s.)

To the average reader, the theory of probability is one applied to the solution of problems of chance and random selection. In the field of social medicine, the statistical techniques developed from the fundamental concepts of probability are part of the essential equipment. In all branches of medicine the use of statistical methods in weighing evidence is becoming daily more common, and indeed statistics have become almost a fetish in socio-medical writings: yet rarely does the user of such techniques examine their philosophical foundation. But to the mathematician, the definition of the fundamental concepts and the nature of the system of logic governing the development of statistical thought are of primary importance. Consequently modern works on the theory of probability concern themselves more and more with the philosophical basis, with definitions of concepts and axioms, and with the fundamentals of mathematical logic from which statistical theory springs. Once these have been accepted, the problems which arise can only be variations, more or less complex, on a limited number of themes: the application of the multinomial expansion, the methods of partition, and the other necessary techniques do not offer much scope for novelty. On the other hand, the last word has by no means been said in the approach to a complete and satisfactory determination of the fundamentals of statistical theory.

The many and varied approaches to probability are customarily grouped in two broad categories, objective and subjective, and any new theory is at once judged and placed in the appropriate category and its sub-division of axiomatic or non-axiomatic, numerical or non-numerical. In such an assessment the main criteria of judgement are the attitudes adopted towards degrees of belief, towards the frequency approach, and towards Bayes' theorem. *Probability and the Weighing of Evidence* is an example of such a development towards a "new" theory. In the first chapter the author gives a brief summary of the basic principles of each of the theories so far propounded, and shows the relation which it bears to its main school, objective or subjective. He declares his own adherence to the axiomatic subjective school with its roots in the concept of probability as a degree of belief. He accepts in the main Koopman's non-objective theory (followed by other statisticians in America and by Bartlett in Great Britain) which attempts to overcome one of the major difficulties of the objective method. This is the difficulty of assessing a degree of belief—fundamentally a personal psychological reaction—in terms of numerical quantities which can be subjected to analytical techniques. Koopman's approach develops two parallel theories, one based on the degrees-of-belief concept, the other on a frequency approach for the solution of problems in objective probability; and he accepts two independent theories as a basis for his method. The author of this book, however, allows finally only one theory, and makes the assumption that the principles governing the development of the abstract subjective theory will apply equally to the objective numerical approach.

Thus the new theory propounded in this book introduces many points open to criticism, for in its attempt to escape the difficulties of the objective approach many assumptions are made the validity of which may be doubted. Firstly, it becomes a basic assumption that probabilities conceived as degrees of belief must be capable of being interpreted as

numerical entities. Secondly, the author's definition of "equally probable"—essential to the abstract theory as a counterpart to the definitions necessary to the numerical theory—using the example of a "perfect pack of cards" has all the disadvantages of the "circular" definition in the classical frequency approach. Inevitably also confusion is created by the two mathematical languages required in the simultaneous development of two separate approaches in Chapters 2 and 3: and this confusion is more confounded in the attempt made in Chapter 4 to reconcile these principles into one unified theory applicable to the problems of probability.

Whatever the reader's feelings as to the validity of the fundamental theory, there are many other comments to be made on the chapters on the practical solution of problems. The axiomatic subjective method necessitates the use of inverse probability beginning with the application of Bayes' theorem. This theorem is not, as the author implies, rejected by statisticians adopting a frequency approach: indeed, in certain fields, particularly in genetics, where prior probabilities can be specified, it is of great value. But where the statement of Bayes' theorem leads to an assumption of prior unknown probabilities of hypotheses, where in fact Bayes' postulate is used, adherents of the frequency school reject further use of the theorem. Often the author does not clearly distinguish between the two. Both the implications behind the probability of a hypothesis and the objections to it should be fairly stated, as also should those involved in the probability distribution function of a hypothesis.

The "plausibility" factor which the author derives in applying Bayes' theorem to the weighing of evidence is justifiably used to assess evidence, and few statisticians would object to this. But the conflict of inverse probability arises again when, after summing all evidence, "odds against the original hypothesis" are calculated by subtraction. Nevertheless it might be of some benefit to judicial processes if all juries were taught addition of the weight of evidence according to the principles suggested by this theory.

There are many other comments and criticisms, too numerous to detail, which could be made. Since the author has stated his intention to omit relevant proof when possible, much cannot be said about the gaps in the mathematical argument: but even where such argument is included, it lacks coherence and lucidity in many places, as for instance in the approach to chi-square (χ^2) or in some of the worked examples in the later chapters. Many readers, too, will join issue with the author's suggestion of curve-fitting by eye in the attempt to find a mathematical description of the distribution function. The first chapter, with its analysis of the many existing approaches to the theory of probability, is one of the best features of this book, and the lay-out of each chapter, giving comments on the development proposed, is good.

It is difficult, however, to decide to whom this book would be of much value. To the advanced statistician the abstract theory has been stated more clearly by Koopman, Carnap, and others, and the mathematical chapters would be quite inadequate. To the more humble seeker after statistical knowledge the mathematical argument would prove incomplete and insufficiently developed, whilst any acceptance of the fundamental theory without investigation of the argument would be unprofitable or even dangerous. And to the non-mathematical worker in social medicine those sections which would be comprehensible might be more easily obtained from other sources.

L. Stein