Studying seasonality by using sine and cosine functions in regression analysis

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Abstract

Study objective—A statistical test that allows for adjustment of confounding can be helpful for the study of seasonal patterns. The aim of this article is to supply a detailed description of such a method. An example of its application is given.

Design—A statistical test is presented that retains the information on the connection of time periods by describing the seasonal pattern as one sine and one cosine function. Such functions can be included into a regression model. The resulting form of the seasonal pattern follows a cosine function with variable amplitude and shift.

Main results—The test is shown to be applicable to test for seasonality. Not only one cosine function per time period, but also a mixture of cosine functions can be used to describe the seasonal pattern. Adjustment for confounding effects is possible.

Conclusions—This method for studying seasonal patterns can be applied easily in a regression model. Adjusted prevalences and odds ratios can be calculated.

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Many studies have been published that concerned seasonal variation, for instance in births,\(^1\) early pregnancy loss,\(^2\) and in congenital malformations.\(^3\) Whether a seasonal pattern exists can be studied in several ways. In this article we demonstrate a method that allows for adjustment of confounding. The first part is a general approach of studying seasonal patterns. In the second part we show a more detailed description of the method by means of mathematical functions. Subsequently, we give an application of this method using fictitious data. As an example we use the study of seasonality in anencephaly frequency at birth.

Studying seasonality, a general approach

To study the seasonal variation in congenital malformations, data analysis can be performed in successive stages. The first step is to calculate and plot the prevalence of malformations at birth per month. Then confidence intervals surrounding the monthly prevalences can be calculated and added to the figure. From this information, it can be inferred whether there are differences in malformations per month and whether there are differences between the months. In the same way, clusters of months can be formed and compared with other clusters of months. If confounding is presumed to occur, the next step is to adjust for such confounding effects. One way to perform this is by means of stratification so that insight can be gained into whether prevalences differ between months or clusters of months after adjustment for confounding. In this phase, problems may occur if several confounding factors are present. Adjustment for their effects simultaneously by means of stratification will often lead to small numbers of observations per month and thus to imprecise estimations of the prevalences. None the less, these preliminary phases of analysis will provide the first indications of whether there is a specific seasonal pattern in malformations. Rough evidence of such a pattern warrants a statistical test. In addition, a method is necessary that allows for adjustment of the effects of several confounders simultaneously.

We focus on the question of whether there is a seasonal pattern in malformations during the course of a year, without paying attention to changes between years. To test for seasonality, a \(\chi^2\) test can be used to detect any departure from a uniform distribution. A more specific test should take into account the connection between time periods such as months or weeks. The method of Edwards\(^4\) tests whether frequencies follow a sine function over 12 months. Also adaptations of the Edwards’ test are suitable, for instance the one of Cave and Freedman\(^1\) to test a bimodal seasonal pattern over 12 months, of Walter and Elwood,\(^5\) which can be used in the case of unequal populations at risk, of Roger\(^6\) for small sample sizes, and of Jones \textit{et al.}\(^7\) for an arbitrary shape of the seasonal effect. The non-parametric Hewitt’s test\(^8\) or its adaptation for other than six month periods by Rogerson\(^9\) can also be applied, but they are less powerful than parametric tests. A Kolmogorov-Smirnov type statistic of Freedman\(^10\) has a better power than the \(\chi^2\) test and the Hewitt’s test in samples of moderate size. None of these tests allows for adjustment of confounding effects, except the method of Jones \textit{et al.}.\(^8\) Some of them, including the latter, require special software. Moreover, the test of Jones \textit{et al.} can only be used for—usually rare—events that follow a Poisson distribution. Therefore another test that allows for adjustment of confounding and that can be performed by widely available statistical computer programs is warranted.

In epidemiological practice, multivariate analysis techniques are commonly used to adjust for confounding. Linear regression analysis is often performed if the dependent variable has a normal distribution. In studies on seasonality in malformations, the dependent
variable is likely to be dichotomous, for example anencephaly is either present or it is not. In such a case, logistic regression analysis can be used.

To test whether congenital malformations are seasonally distributed, one sine and one cosine function can be introduced into the regression model. This results in a pattern following a cosine function with variable amplitude and shift. Depending on the hypothesis being tested, the period of the cosine function can be one year, half a year or shorter. The maximum likelihood method estimates the regression coefficients for the best fitting regression line. The amplitude and the amount of shift of the cosine function can be calculated from the regression coefficients. For each time period, the probability of a malformation and the odds ratios can be calculated using the logistic regression model. Such a method was applied before, for instance by Bound et al.3 and Woodhouse et al.,12 but not fully explicated. Therefore this paper will give a detailed description and an example. For more advanced models in time series we refer to Fahrmeir and Tutz.13

Detailed description of the method
A linear regression model can be developed to analyse seasonality in congenital malformations. Generally speaking, such a model will have the following form:

\[ y = \beta_0 + \beta_{\text{anomaly}} \times \text{season} + \beta_{C_1} \times C_1 + \ldots + \beta_{C_N} \times C_N \]

where \( \beta \) is the intercept and \( C \) indicates a confounder; \( y \) is a continuous variable related to the presence of a congenital malformation and is normally distributed or can be transformed into such a distribution. An example of such a variable is the level of \( \alpha \)-fetoprotein. In many studies on congenital malformations, the outcome parameter is defined as the probability of malformation. This probability can be modelled in a logistic regression model such as:

\[ \ln \left( \frac{P}{1-P} \right) = \beta_0 + \beta_{\text{anomaly}} \times \text{season} + \beta_{C_1} \times C_1 + \ldots + \beta_{C_N} \times C_N \]

where \( P \) is the probability of a malformation, for instance the probability of anencephaly. To define the variable “season” in these models, it is hypothesised that the seasonal pattern under study follows a cosine function with variable amplitude and horizontal shift. In this cosine function, two periods must be defined: (a) the time period that defines the measure of malformation, for example, “month” in “the probability of an anencephalus birth per month” and (b) the period described by one cosine function. As an example we take “month” as the time period under study, and “one year” as the period of the cosine function. The cosine function can be described as:

\[ f(t) = \alpha \times \cos \left( \frac{2\pi t}{T} - \theta \right) \]

One value \( t \) is retrieved. If \( \beta_1/\beta_2 > 0 \), then \( t > 0 \) and indicates the first extreme; the other extreme value is found at \( t + T/2 \). If \( \beta_1/\beta_2 < 0 \), then \( t < 0 \); the extreme values are found at \( t + T/2 \) and at \( t + T \). If \( \beta_1 > 0 \), the first extreme is a maximum and the second a minimum; if \( \beta_1 \leq 0 \), the first extreme is a minimum and the second a maximum.
Application to data

As an example we applied this method for studying seasonality to data from anencephalus births and total births described by Walter and Elwood* (table 1). In their article they presented the results of several tests of seasonality: the method of Edwards using only case frequencies or using adjusted frequencies, their own method assuming months of equal length or exact month lengths and Hewitt’s non-parametric test. They found that neither Edwards’s test using frequencies nor Hewitt’s test detected a seasonal pattern. The other three methods did find a seasonal pattern with the maximum prevalence of anencephaly in late December.

We used sine and cosine functions in a logistic regression analysis to test for seasonality in the prevalence of anencephaly. Firstly, we tested whether there was a seasonal pattern with one maximum level and one minimum level per year—that is, a cosine function with a period of 12 months. Secondly, we studied whether the seasonal pattern in the prevalence of anencephaly was better described by a cosine function with a period of six months (that is, \( T = 6 \)), or, thirdly, by a mix of one cosine function with a period of 12 months and one with a period of six months. In figure 1 the cosine functions are shown. In table 2 the logistic regression models and their results are presented: regression coefficients, maximum and minimum levels, likelihood ratio test results, and deviances.

Firstly, the test of a seasonal pattern with one maximum per year. The time period is the “month” and the period of the cosine function is “one year”. We also found a seasonal pattern. The maximum prevalence was observed in late December (\( t_{\text{max}} = 0.27 \)) and the minimum in June (\( t_{\text{min}} = 0.27 + T/2 = 6.27 \)), in agreement with the results described by Walter and Edwards. The pattern could be described by a
cosine function with an amplitude of 0.070 and a shift of 0.14 (≈ 2π × 0.27/T) radials. Secondly, the test of seasonal pattern that could be described by a cosine function with a period of six months showed no better fit than the former model. Then we tested whether the model using the cosine function with a period of 12 months could be improved by adding a second cosine function to the model with a period of six months. From figure 2 and the likelihood ratio test result in table 2 it can be derived that this extra cosine function improved the model. This model with a mix of cosine functions indicated the highest peak about in February and the lowest through about in June. The goodness of fit statistic for this model is given by a deviance of 15.39, df=7, p=0.03, which shows a statistically lack of fit. This clear indication of overdispersion was added as a shift of 0.14 (= 2π × 0.27/T) radials. This model with a mix of cosine functions fitted the data of anencephaly per 100 000 births, crude and after adjustment for sex (fictitious data). Dotted line, crude cosine function with period of 12 months; inverted line, sex adjusted cosine function with period of 12 months; square symbols, prevalence in total population; solid line, sex adjusted cosine function with period of 12 months, inverted line, crude cosine function with period of 12 months; triangle symbols, prevalence among boys; crosses, prevalence among girls.

Figure 2  Probability of a child with anencephaly (P), expressed as the prevalence of anencephaly per 100 000 births, crude and after adjustment for sex (fictitious data). Dotted line, crude cosine function with period of 12 months; square symbols, prevalence in total population; solid line, sex adjusted cosine function with period of 12 months, inverted line, crude cosine function with period of 12 months; triangle symbols, prevalence among boys; crosses, prevalence among girls.

The maximum extreme was found in June (tmax = 0.54 + 7π ÷ 6.54). This results in a cosine function with an amplitude of 0.43 and the shift of 2π × 6.54/T = 3.42 radials (fig 2). Thus after adjustment for the fictitiously introduced sex distribution, we found a totally opposite seasonal pattern with the maximum prevalence observed in June and the minimum in December.

We conclude that it is possible to test for seasonal patterns by means of applying sine and cosine functions into regression analysis. Not only a period of, for instance, 12 or 6 months can be described by such a cosine function, but also a mixture of cosine functions is possible. Moreover, this way of analysing seasonality allows for adjustment of confounding effects.

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